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FOREWORD

The basic equations required to acquire an understanding of the autopilot, roll system and guidance computer for a homing missile are derived. How the use of these equations lead to methods for analysis and design of the systems is shown.

The factors that affect the design are enumerated and the most important ones, explained in detail.

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LIST OF SYMBOLS

$$A = 1481 \lambda S M^2 \frac{57.3}{mv} C_{N\alpha}$$

or $1481 \lambda S M^2 \frac{57.3}{mv} C_{Y\beta}$

$$B = 1481 \lambda S M^2 \frac{57.3}{mv} C_{m\dot{\alpha}}$$

or $1481 \lambda S M^2 \frac{57.3}{mv} C_{Y\dot{\beta}}$

$$C = 1481 \lambda S dM^2 \frac{57.3}{I_y} C_{N\alpha}$$

or $1481 \lambda S dM^2 \frac{57.3}{I_x} C_{Y\beta}$

$$E = 1481 \lambda S dM^2 \frac{57.3}{I} C_{m\dot{\alpha}}$$

or $1481 \lambda S dM^2 \frac{57.3}{I} C_{m\dot{\beta}}$

$$F = 1481 \lambda S dM^2 \frac{57.3}{I_y} \frac{b}{2v} C_{L\dot{\beta}}$$

$$G = 1481 \lambda S dM^2 \frac{57.3}{I_x} C_{L\dot{\alpha}}$$

$$H = 1481 \lambda S dM^2 \frac{57.3}{I_x} C_{L\dot{\beta}}$$

$$I = 1481 \lambda S dM^2 \frac{57.3}{I_x} C_{L\dot{\alpha}}$$

$$J = 1481 \lambda S M^2 \frac{57.3}{mv} C_{Y\dot{\beta}}$$

$$N = 1481 \lambda S dM^2 \frac{57.3}{I_x} C_{m\dot{\beta}}$$

$C_{N\alpha}, C_{N\dot{\alpha}}$ = pitch force derivatives

$C_{Y\beta}, C_{Y\dot{\beta}}, C_{Y\ddot{\beta}}$ = yaw force derivatives

$C_{m\alpha}, C_{m\dot{\alpha}}$ = pitch moment derivatives

$C_{L\alpha}, C_{L\dot{\alpha}}, C_{L\ddot{\alpha}}$ = yaw moment derivatives

$C_{L\beta}, C_{L\dot{\beta}}, C_{L\ddot{\beta}}$ = roll moment derivatives

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$E(X)$ = modulus of elasticity at any point X

$F(t)$ = arbitrary function of time

F_T = force input at tail

$G(s)$ = forward transfer function

$G_a(s)$ = accelerometer response

$G_f(s)$ = filter transfer function

$G_r(s)$ = rate gyro response

$G_g(s)$ = guidance computer transfer function

$G_c(s)$ = control surface servo transfer function

$H(s)$ = feedback transfer function

I_x = roll moment of inertia

I_x, I_y, I_z = pitch, yaw moment of inertia

$I(X)$ = area moment of inertia of the beam cross-section

J = generalized roll moment of inertia

K = gain for any system

K_1, K_2, K_3 = autopilot gains

K_4, K_5, K_6 = roll system gains

K_7, K_8, K_9 = servo component gains

M_T = torque input at tail

M_a = aerodynamic moment

M_1 = generalized mass of first bending mode

M = mach number

$N = \frac{A V_R}{V_{11}}$ = navigation ratio

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$$T_{ij} = \frac{E}{AE - BC}$$

$V = V_M$ " missile velocity

V_R " relative closing velocity, missile to target

W " missile weight

d " body diameter

f " aerodynamic force

δ " control surface deflection

\vec{j}, \vec{k} " unit vectors

$m(x)$ " mass distribution along length of missile

\ddot{x} " axial acceleration

\ddot{y} " acceleration normal to missile centerline

\ddot{z} " called for acceleration

$\ddot{\beta}$ " acceleration due to body bending

$\ddot{\eta}$ " accelerometer output

$\ddot{\eta}_{noise}$ " acceleration due to noise

\dot{P} " rate of rotation about X axis

q " dynamic pressure

\dot{q} " rate of rotation about Y axis

\dot{r} " radome error slope

S " body cross sectional area

u " velocity along X axis

v " velocity along Y axis

w " velocity along Z axis

$\dot{\psi}$ " rate of rotation about Z axis

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- X = distance along missile centerline
- X_T = location of tail surface hinge line
- X_a = location of accelerometer
- X_r = location of rate gyro
- α = angle of attack
- β = side slip angle
- β = angle between seeker centerline and missile centerline
- δ = angle between velocity vector and reference
- δ = control surface deflection to produce roll torque
- ϵ = geometrical tracking error angle (angle between seeker nutation axis and line of sight)
- ϵ_1 = error due to radome refraction
- ϵ = receiver output
- ϵ_{noise} = noise on receiver output signal
- ζ = damping coefficient for second order system (% critical damping)
- θ = angle between seeker centerline and reference
- θ_{comp} = computed angle of twist at station X for body torsional mode
- Λ = effective navigation ratio
- λ = static pressure ratio (static pressure at altitude to that at S.L.)
- σ = angle between line of sight to target and reference
- T = time constant
- T_h, T_a = time constants for autopilot
- T_v = servo valve time constant
- T_f = filter time constant

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- $\dot{\phi}$ = roll rate gyro output
- ϕ = roll angle
- ϕ_n = mode function for the mode
- ϕ_n^0 = normalized mode function for the mode
- γ = angle between missile centerline and reference
- $\dot{\psi}$ = pitch rate gyro output
- $\dot{\eta}$ = rate of body bending at rate gyro station
- ω_b = natural frequency for second order system
- ω_η = missile first bending mode natural frequency
- ω_τ = missile first torsional mode natural frequency

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1. INTRODUCTION:

This report has been written to serve possibly two purposes. First to acquaint people not familiar with the autopilot, roll system or homing guidance system with some of the problems involved in their design and suggest methods of approach for solving these problems, and second to present under one cover some of the analytical work that is basic in the design of the systems. There are therefore both elementary and detailed information regarding the systems and methods for analysis and synthesis. No mention is made of any analogue computer techniques since this is familiar ground for everyone. The information in this report should provide useful background material prior to the campaign on the pot setter.

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2. AUTOPILOT DESIGN:

The autopilot will be defined as the complete system for which the input and the output are the following:

- Input - Command acceleration in any direction normal to the missile centerline.
Output - Resultant acceleration of the missile.

The airframe characteristics, the control systems, and the sensing instruments, if any, are involved.

2.1 Derivation of Transfer Functions For The Airframe.

The basic design of the autopilot can be carried on by considering the system to have three degrees of freedom (1) missile rotation in pitch, (2) missile c.g. translation normal to the body in the pitch plane and (3) control surface deflection to produce motion in pitch plane.

Two additional degrees of freedom, (1) rotation of missile about longitudinal axis and (2) tail surface deflection to produce roll, are included in the final analysis to check the compatibility of the autopilot design with the roll control system in the presence of aerodynamic coupling. The equations of motion and the transfer function for this analysis are developed in the section on roll system design.

The angles for the single plane analysis are shown in Figure 1. The sign convention is also shown.

The Angles are the Following:

- α = Angle of attack = angle between missile centerline and velocity vector.
 γ = Angle between the velocity vector and the reference.
 ψ = Angle between the missile centerline and the reference.
 δ = Angle between the control surface and the missile centerline.

The relationship between the rate of rotation of the velocity vector and the acceleration of the c.g. normal to the missile body will be derived first.

Let \vec{J} be a unit vector along \vec{V} and let \vec{K} be a unit vector \perp to \vec{V} and positive in the direction for increasing γ . (See Figure 1), then the velocity of the c.g. is $\vec{V} = V \vec{J}$ and the acceleration of the c.g. is

$$\frac{d\vec{V}}{dt} = \dot{V} \vec{J} + V \frac{d\vec{J}}{dt} \quad \text{but, } \frac{d\vec{J}}{dt} = \dot{\gamma} \vec{K} \quad (\dot{\gamma} \text{ in Rad/Sec})$$

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therefore,

$$2.1-1 \quad \frac{d\vec{V}}{dt} = \dot{V} \hat{j} + V \dot{\delta} \hat{k} \quad (ft/sec^2)$$

$V \dot{\delta}$ is therefore the acceleration of c.g. normal to the velocity vector.

The acceleration of the c.g. normal to the missile body is defined by n_z (g's) and along the missile axis, by n_x (g's)

The relationship between the accelerations with respect to the missile axis and

$$V \dot{\delta} \quad \text{is} \quad \frac{V \dot{\delta}}{32.2} = n_z \cos \alpha + n_x \sin \alpha$$

if $\dot{\delta}$ is in deg/sec and $n_x \sin \alpha$ is assumed to be negligible compared to $n_z \cos \alpha$

$$2.1-2 \quad \frac{V \dot{\delta}}{1045} = n_z \quad \text{or} \quad \dot{\delta} \approx \frac{1845}{V} n_z$$

This relationship is used frequently in the following sections.

The aerodynamic forces normal to the missile body n_z is a function of α and z

$$W n_z = f(\alpha, z) \quad \text{where } W = \text{weight of missile in pounds}$$

$$\text{or } W dn_z \approx \frac{\partial f}{\partial \alpha} d\alpha + \frac{\partial f}{\partial z} dz \quad (\text{slug ft/sec}^2)$$

Assuming the partials to be constants and integrating

$$2.1-3 \quad W n_z = \frac{\partial f}{\partial \alpha} \alpha + \frac{\partial f}{\partial z} z$$

Similarly for the aerodynamic moments.

$$2.1-4 \quad I \ddot{\theta} = \frac{\partial M_a}{\partial \alpha} \alpha + \frac{\partial M_a}{\partial z} z \quad (\text{slug ft}^2/\text{sec}^2)$$

I = pitch moment of inertia (slug ft.²).

The moment is also a function of $\dot{\theta}$ but this term is neglected since the control system of the type proposed with rate gyro feedback makes it a negligible factor.

The partial derivatives $\frac{\partial f}{\partial \alpha}$, $\frac{\partial f}{\partial z}$, $\frac{\partial M_a}{\partial \alpha}$ and $\frac{\partial M_a}{\partial z}$ are one form of the stability derivatives. Normally they are written in the form

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$$C_{N\alpha} = \frac{1}{qS} \frac{\partial f}{\partial \alpha} \left(\frac{1}{\text{deg}} \right)$$

$$C_{Nz} = \frac{1}{qS} \frac{\partial f}{\partial z} \left(\frac{1}{\text{deg}} \right)$$

$$C_{m\alpha} = \frac{1}{qSd} \frac{\partial M_a}{\partial \alpha} \left(\frac{1}{\text{deg}} \right)$$

$$C_{mz} = \frac{1}{qSd} \frac{\partial M_a}{\partial z} \left(\frac{1}{\text{deg}} \right)$$

$$q = \text{dynamic pressure} = \frac{1}{2} \rho V^2 \quad (\text{slug/sec}^2 \text{ ft.}) \\ = 1481 \lambda M^2$$

where ρ = density of air (slug/ft.³)

λ = ratio of static pressure at altitude to that at sea level.

M = Mach No.

S = reference area (normally body area) (ft.²) = .994 ft.² for Tartar

d = reference moment arm (normally body diameter) (ft.) = 1.125 ft. for Tartar

For purposes of analysis the force and moment equations 2.1-3 and 2.1-4 are put in the form

$$\begin{aligned} 2.1-5 \quad \delta &= A\alpha + Bz \\ \ddot{\psi} &= C\alpha + Ez \end{aligned}$$

where all angles are in degrees.

The relationship between these coefficients and the original stability derivatives can be easily deduced.

$$\begin{aligned} A &= \frac{(184.5)(1481) \lambda S M^2 C_{N\alpha}}{VW} \\ B &= \frac{(184.5)(1481) \lambda S M^2 C_{Nz}}{VW} \\ C &= \frac{(57.3)(1481) \lambda S d M^2 C_{m\alpha}}{I} \\ E &= \frac{(57.3)(1481) \lambda S d M^2 C_{mz}}{I} \end{aligned}$$

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The values of $C_{m\alpha}$ and C_{m_i} must be the values for the particular c.g. location considered. If these coefficients are available for a reference c.g. location and have the values $C_{m_{0\alpha}}$ and $C_{m_{0i}}$, the values at any other c.g. location are given by.

$$C_{m\alpha} = C_{m_{0\alpha}} + \frac{X-X_0}{d} C_{N\alpha}$$
$$C_{m_i} = C_{m_{0i}} + \frac{X-X_0}{d} C_{N_i}$$

where X_0 is the reference c.g. and X is the desired c.g. Both these quantities are in inches and measured from Station 0 which is at or near the nose of the missile. d is the reference moment arm in inches (13.5 inches for the Tartar).

The transfer functions for the airframe can be derived from equation 2.1-5. Assuming all initial conditions are zero, the operational form of these equations are.

2.1-6

$$s\delta = A\alpha + Bz$$
$$s^2\psi = C\alpha + Ez$$

from the definition of the angles

$$\alpha = \psi - \delta$$

substituting

$$s\delta - A\psi + A\delta = Bz$$
$$s^2\psi - C\psi + C\delta = Ez$$

or

$$(s+A)\delta - A\psi = Bz$$
$$C\delta + (s^2-C)\psi = Ez$$

By use of Cramers Rule

$$\frac{\delta}{z} = \frac{\begin{vmatrix} B & -A \\ E & (s^2-C) \end{vmatrix}}{\begin{vmatrix} s+A & -A \\ C & (s^2-C) \end{vmatrix}}$$

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$$s\ddot{x} = \frac{Bs^2 - BC + AE}{s^2 + As - C}$$

$$2.1-7 \quad \ddot{x}/s = -\left(\frac{AE-BC}{C}\right) \frac{1 + \frac{Bs^2}{AE-BC}}{-s^2/c - A/s + 1}$$

$$s\ddot{x}/s = \frac{\begin{vmatrix} s+A & B \\ C & E \end{vmatrix}}{s^2 + As - C} = \frac{Es + AE - BC}{s^2 + As - C}$$

$$2.1-8 \quad \ddot{x}/s = -\left(\frac{AE-BC}{C}\right) \frac{1 + \frac{E}{AE-BC} s}{-s^2/c - A/s + 1}$$

Substituting n_3 for \ddot{x} equation 2.1-7 is

$$2.1-9 \quad n_3/s = -\frac{V}{1845} \left(\frac{AE-BC}{C}\right) \frac{1 + \frac{Bs^2}{AE-BC}}{-s^2/c - A/s + 1}$$

For the Tartar missile the values of A and C will vary with angle of attack. To facilitate the analysis, however, constant values for these coefficients are used. The analysis will be valid for the angle of attack region for which the coefficients were picked.

The values for these coefficients for the Tartar missile are given in the following table for two typical flight conditions.

Case Number	Flight Condition	α'	A	B	C	E
1	M 1.5	6°	1.75	.845	-.32	-.391
2	S.I.	18°	2.2	.845	.60	-.391
3	M 2.0	6°	.65	.226	-.1	-.141
4	30,000 ft. Altitude	18°	1.05	.226	.4	-.141

These values are for a combined plane maneuver, i.e., the resultant angle of attack and g's are in a plane 45° from the plane of the wings.

Substituting the values from the table in equations 2.1-8 and 2.1-9 these equations became.

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Case

1. $\frac{\dot{\eta}_1}{\eta_1} = -20.6 \frac{1 + 1.505s}{0.3s^2 + 0.0546s + 1}$
 $\frac{\dot{\eta}_1}{\eta_1} = \frac{-20.6}{1845} \frac{1 - 0.001285s^2}{0.00235s^2 + 0.0546s + 1}$

2. $\frac{\dot{\eta}_1}{\eta_1} = 9.9 \frac{1 + 1.495s}{-0.125s^2 - 0.0375s + 1}$
 $\frac{\dot{\eta}_1}{\eta_1} = \frac{9.9}{1845} \frac{1 - 0.001685s^2}{-0.125s^2 - 0.0375s + 1}$

3. $\frac{\dot{\eta}_1}{\eta_1} = -9.9 \frac{1 + 1.445s}{s^2 + 1.655s + 1}$
 $\frac{\dot{\eta}_1}{\eta_1} = \frac{-9.9}{1845} \frac{1 - 0.002465s^2}{s^2 + 1.655s + 1}$

4. $\frac{\dot{\eta}_1}{\eta_1} = 36.8 \frac{1 + 1.957s}{-1.25s^2 - 1.265s + 1}$
 $\frac{\dot{\eta}_1}{\eta_1} = \frac{36.8}{1845} \frac{1 - 0.01535s^2}{-1.25s^2 - 1.265s + 1}$

These equations are, in general, of the form

$$\frac{\dot{\eta}_1}{\eta_1} = K \frac{1 + T_1 s}{1 + T_2 s + T_3 s^2}$$

and

$$\frac{\dot{\eta}_2}{\eta_2} = \frac{K}{1845} \frac{1}{1 + 2.15 + T_3 s^2}$$

$$\frac{\dot{\eta}_3}{\eta_3} = \frac{K}{1845} (1 + T_4 s)$$

The significant factors that are apparent from these equations are:

1. The airframe is essentially a second order system.
2. The airframe by itself can be divergently unstable for certain flight conditions and angles of attack. For cases 2 and 4 the roots of the characteristic equation have positive real parts.
3. The zero frequency gain, K, varies in magnitude and polarity with flight condition and angle of attack.
4. $\dot{\psi}$ is approximately the derivative of η_2 . That is it can be used as a measure of the rate of η_2 . $\dot{\psi}$ feedback can therefore be used to provide damping for the system.

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2.2 Form of Control Equations

The control equation can have any number of forms. The most common are the following:

$$2.2-1 \quad \dot{\lambda}_c = -K_1 n_c \quad (\text{Type A})$$

$$2.2-2 \quad \dot{\lambda}_c = -K_1 n_c + K_2 \dot{\psi}' \quad (\text{Type B})$$

$$2.2-3 \quad \dot{\lambda}_c = K_1 (\dot{n}' - n_c) + K_2 \dot{\psi}' \quad (\text{Type C})$$

$$2.2-4 \quad \dot{\lambda}_c = \frac{K_1 (\dot{n}' - n_c) + K_2 \dot{\psi}' + K_3 \ddot{\psi}'}{s} \quad (\text{Type D})$$

where λ_c is the command wing deflection and n_c is the command acceleration; and the prime represents the instrument outputs.

Block diagrams of the complete autopilot system using these control equations are shown in Figure 2 - (a), (b), (c) and (d). These systems will be explained in detail after the closed loop transfer functions for each are derived.

The closed loop transfer functions can be derived by using the equations from section 2.1. The following assumptions will be made for the servo and instrument responses.

$$G_r(s) = 1$$

$$G_n(s) = 1$$

$$G_s(s) = 1 \quad \text{For Types A, B, C} \\ = \frac{1}{s} \quad \text{For Type D}$$

These assumptions do not invalidate the general conclusions that can be drawn for these systems. The actual choice of gains however cannot be made without consideration of these transfer functions.

For the type A system, substitution of equation 2.1-9 into equation 2.2-1 results in

$$2.2-5 \quad \frac{n_3}{n_c} = \frac{K_1 V}{1845} \left(\frac{AE - BC}{C} \right) \frac{1 + \frac{Bs^2}{AE - BC}}{-\frac{s^2}{C} - \frac{A}{C}s + 1}$$

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For the remaining types of control systems, equation 2.1-6 is used with the control equations.

For type B the derivation is as follows:

$$(s+A)\psi - A\psi - B\tau = 0$$

$$C\psi + (s^2 - c)\psi - E\tau = 0$$

$$K_2\psi - \tau = K_1 n_c$$

$$\frac{\psi}{n_c} = \begin{vmatrix} 0 & -A & -B \\ 0 & s^2 - c & -E \\ K_1 & K_2 s & -1 \end{vmatrix} / \begin{vmatrix} s+A & -A & -B \\ C & s^2 - c & -E \\ 0 & K_2 s & -1 \end{vmatrix}$$

$$\frac{\psi}{n_c} = \frac{K_1(AE - BC + BS^2)}{-K_2s(-ES - EA + BC) - (BS^2 + AS^2 - cS)}$$

2.2-6 using $\tau = \frac{V}{1845} \dot{\psi}$

$$\frac{\psi}{n_c} = \frac{V}{1845} \left(\frac{AE - BC}{C} \right) - \frac{K_1 \left(1 + \frac{BS^2}{AE - BC} \right)}{\left(\frac{E}{C} + \left(\frac{A}{C} + \frac{K_2 E}{C} \right) s + 1 + \frac{K_1}{C} (AE - BC) \right)}$$

For type C the derivation is as follows:

$$(s+A)\psi - A\psi - B\tau = 0$$

$$C\psi + (s^2 - c)\psi - E\tau = 0$$

$$\frac{K_1 V}{1845} s\psi + K_2 \psi - \tau = K_1 n_c$$

$$\frac{\psi}{n_c} = \begin{vmatrix} 0 & -A & -B \\ 0 & s^2 - c & -E \\ K_1 & K_2 s & -1 \end{vmatrix} / \begin{vmatrix} s+A & -A & -B \\ C & s^2 - c & -E \\ \frac{K_1 V}{1845} s & K_2 s & -1 \end{vmatrix}$$

2.2-7

$$\frac{\psi}{n_c} = \frac{1 + \frac{BS^2}{AE - BC}}{\left(\frac{K_1 V B}{1845} - 1 \right) s^2 + \frac{(K_2 E - A)}{\frac{V}{1845} K_1 (AE - BC)} s + \frac{K_2}{\frac{V}{1845} K_1} + \frac{C}{\frac{V}{1845} K_1 (AE - BC)} + 1}$$

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For type D the derivation is as follows:

$$(s+A)\psi - A\psi - Bz = 0$$

$$C\psi + (s^2 - c)\psi - Ez = 0$$

$$\frac{K_1 V}{1845} s\ddot{\psi} + (K_2 s + K_3 s^2)\psi - sz = K_1 n_c$$

$$\frac{\ddot{\psi}}{n_c} = \begin{vmatrix} 0 & -A & -B \\ 0 & s^2 - c & -E \\ K_1 & K_2 s + K_3 s^2 & -s \end{vmatrix} = \begin{vmatrix} s+A & -A & -B \\ c & s^2 - c & -E \\ \frac{K_1 V}{1845} s & K_2 s + K_3 s^2 & -s \end{vmatrix}$$

$$2.2-8 \quad \ddot{\psi}/n_c = \frac{1 + \frac{Bs^2}{K_1(AE-BC)}}{\frac{-s^3}{\frac{K_1 V(AE-BC)}{1845}} + \frac{(EK_2 + \frac{BK_1 V}{1845} - A)s^2}{\frac{K_1 V(AE-BC)}{1845}} + \frac{[(AE-BK_3)K_2 + EK_2 + E]s}{\frac{K_1 V(AE-BC)}{1845}} + \left[1 + \frac{K_2}{\frac{K_1 V}{1845}}\right]}$$

For purposes of examining the $\ddot{\psi}/n_c$ transfer functions qualitatively the following approximations will be made

$$AE - BC \approx AE$$

$$\frac{B}{AE-BC} \approx 0$$

The transfer functions for the four types of systems become.

Type A $(z_c = -K_1 n_c)$

$$2.2-9 \quad \ddot{\psi}/n_c \approx \frac{\frac{K_1 V}{1845} \left(\frac{AE}{c}\right)}{\frac{-s^3}{\frac{K_1 V(AE-BC)}{1845}} - \frac{1}{\frac{K_1 V(AE-BC)}{1845}} - \frac{B}{c}s + 1}$$

Type B $(z_c = -K_1 n_c + K_2 \psi)$

$$2.2-10 \quad \ddot{\psi}/n_c \approx \frac{\frac{K_1 V}{1845} \left(\frac{AE}{c}\right)}{\frac{-s^3}{\frac{K_1 V(AE-BC)}{1845}} - \frac{1}{\frac{K_1 V(AE-BC)}{1845}} - \left(\frac{B}{c} + \frac{K_2}{c}E\right)s + 1 + \frac{K_2}{c}}$$

Type C $(z_c = K_1(n' - n_c) + K_2 \psi')$

$$2.2-11 \quad \ddot{\psi}/n_c \approx \frac{-s^2}{\frac{V}{1845} K_1(AE)} + \frac{(K_2 E - A)}{\frac{V}{1845} K_1 AE} s + 1 + \frac{K_2}{\frac{V}{1845} K_1} + \frac{c}{\frac{V}{1845} K_1 AE}$$

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Type D

$$(\tau_c = \frac{K_0(n' - n_c) + K_2 \psi' + K_3 \psi''}{s})$$

2.2-12

$$\eta/n_c = \frac{-s^2 + \frac{EK_3 - A}{K_1VAE} s^2 + \left(\frac{K_2}{K_1VAE} + \frac{K_3}{K_1VAE} + \frac{C}{K_1VAE} \right) s + \left(1 + \frac{K_4}{K_1VAE} \right)}$$

Type A is the simplest form possible. This system can operate only over the range of angle of attack where $\frac{AE}{C}$ is fairly constant and the airframe is stable

by itself. The gain, K_0 , must be made to vary with flight conditions if the zero frequency gain (static gain) is required to be constant. The damping for the system is provided only from the airframe. In general the damping coefficient ξ (fraction of critical damping) is on the order of .1 to .2 for those angles of attack where the $\frac{AE}{C}$ is approximately constant.

Type B uses a rate gyro feedback to improve the damping characteristic of the system. The requirements on $\frac{AE}{C}$, stability of the airframe, and K_1 , are the same as for the Type A system. K_2 can be chosen to produce any ξ desired.

Type C system uses an additional accelerometer feedback. This removes the restriction on variations in $\frac{AE}{C}$ and the stability of the airframe if the gains K_1 and K_2 are chosen properly. If it is desirable to maintain approximately the same speed of response and damping characteristic at all flight conditions, these gains can be made to vary with some measurable quantity which varies in the same manner as the aerodynamic coefficients. Total pressure is such a quantity.

The variation in static gain with variation in Q will depend on the highest value that can be used for K_1 . The restriction on the highest value for K_1 is explained in the next section.

Type D system is similar to that used for the Tartar. In this system a rate feedback control surface servo is used. This results in an additional integration and requires a feedback term proportional to the derivative of the rate gyro signal. The autopilot is now a third order system instead of the second order systems for the other three. All three coefficients for the 3rd order characteristic equation can be controlled by proper choice of K_1 , K_2 and K_3 . The static gain for this system is seen to depend only on $\frac{K_4}{K_1VAE}$. If this term is approximately a constant then the static gain will be approximately constant for all angles of attack and flight conditions. If the term is not approximately a constant, as in the Tartar missile, the ψ' feedback term can be modified to be $-\psi' - \frac{1845}{V_{ar}} \eta'$.

$$\text{Since } \eta' \frac{1845}{V_{ar}} \approx \delta, \quad \psi' - \frac{1845}{V_{ar}} \eta' \approx \alpha$$

This results in a control equation of the form

$$\tau_c = \frac{K_1(n' - n_c) + K_2 \alpha' + K_3 \alpha''}{s}$$

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The closed loop transfer function will be modified slightly and be of the form

$$2.2-12 \quad \frac{H}{H_c} \approx \frac{1}{\frac{-s^3}{K_1 \frac{VAE}{1845}} + \frac{EK_3 - A}{K_1 \frac{VAE}{1845}} s^2 + \left(\frac{K_2}{K_1 \frac{VAE}{1845}} + \frac{C}{K_1 \frac{VAE}{1845}} \right) s + 1}$$

With this form the static gain is approximately unity at all flight condition and for any angle of attack.

Type A and Type B systems are not practical for the Tartar missile since the variations in $\frac{H}{H_c}$ with angle of attack is prohibitive and the airframe is unstable for large angles of attack.

Type C and Type D are possible systems for the Tartar missile. These are the only systems that will be considered in the following sections.

2.3. Stability Analysis.

If the systems were complete as presented in the previous section the only analysis that would be required would be an examination of the roots of the characteristic equation. For the type C system this means the determination of the roots of a quadratic equation, and for type D, the determination of the roots of a cubic equation. In general if the real parts of the roots are negative, the systems are stable. There will be adequate stability if the ratio of the real part to the imaginary part is the tangent of an angle less than 60° . This corresponds to a $\xi \geq .5$.

For the complete system, however, such an analysis is inadequate. The effects of the following factors must still be determined.

- (1). Instrument responses
- (2). Servo response
- (3). Rate limit on the servo
- (4). Expected tolerances on the instruments, circuitry and gains.
- (5). Additional filters for noise or body vibration.

The most satisfactory method of examining these effects has been to examine the open loop transfer function by Nyquist plots. For multiloop systems in general the open loop function will vary with where the loop is opened. The open loop transfer functions for the two systems will be derived first. They will be derived for the

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following case .

- (1). Loop opened at the wing servo
- (2). Loop opened at the rate gyro.
- (3). Loop opened at the accelerometer.

See Figure 3 (a), (b) and (c).

For type C, Case (1)

$$\frac{\dot{\theta}_0}{\dot{\theta}_{in}} = G_s(s) \left[K_2 G_r(s) \left(\frac{\psi}{\dot{\theta}} \right) + K_1 G_a(s) \left(\frac{\eta}{\dot{\theta}} \right) \right]$$

Substituting for $\frac{\psi}{\dot{\theta}}$ and $\frac{\eta}{\dot{\theta}}$ from equations 2.1-8 and 2.1-9

$$2.3-1 \quad \frac{\dot{\theta}_0}{\dot{\theta}_{in}} = -G_s(s) \left(\frac{AE-BC}{C} \right) \frac{\left[K_2 G_r(s) \left(1 + \frac{E}{AE-BC} s \right) + \frac{K_1 V}{1845} \left(1 + \frac{Bs^2}{AE-BC} \right) G_a(s) \right]}{1 - \frac{A}{C} s - \frac{s^2}{C}}$$

For type C, Case (2)

$$\frac{\dot{\theta}_0}{\dot{\theta}_{in}} = G_r(s) K_2 \left(\frac{\psi}{\dot{\theta}} \right) \left[\frac{G_s(s)}{1 - G_s(s) \left(\frac{\eta}{\dot{\theta}} \right) G_a(s) K_1} \right]$$

Since negative feedback is not implied for any of the inner loops considered the negative sign is put on H (S).

$$\frac{\dot{\theta}_0}{\dot{\theta}_{in}} = -K_2 G_r(s) \left(\frac{AE-BC}{C} \right) \frac{\left(1 + \frac{E}{AE-BC} s \right)}{\frac{-s^2}{C} - \frac{A}{C} s + 1} \left[\frac{G_s(s)}{1 - G_s(s) G_a(s) K_1 \left(\frac{V}{1845} \right) \left(\frac{AE-BC}{C} \right) \frac{1 + \frac{Bs^2}{AE-BC}}{-\frac{s^2}{C} - \frac{A}{C} s + 1}} \right]$$

$$2.3-2 \quad \frac{\dot{\theta}_0}{\dot{\theta}_{in}} = -K_2 G_r(s) \left(\frac{AE-BC}{C} \right) \left(1 + \frac{E}{AE-BC} s \right) \left[\frac{G_s(s)}{1 - \frac{A}{C} s - \frac{s^2}{C} + G_s(s) G_a(s) \frac{V}{1845} \left(\frac{AE-BC}{C} \right) K_1 \left(1 + \frac{Bs^2}{AE-BC} \right)} \right]$$

For Type C Case (3)

$$\frac{\dot{\theta}_0}{\dot{\theta}_{in}} = \frac{K_1 G_a(s) \left(\frac{\eta}{\dot{\theta}} \right) (G_s(s))}{1 - G_s(s) K_2 G_r(s) \left(\frac{\psi}{\dot{\theta}} \right)}$$

$$2.3-3 \quad \frac{\dot{\theta}_0}{\dot{\theta}_{in}} = \frac{-K_1 G_a(s) \frac{V}{1845} \left(\frac{AE-BC}{C} \right) \left(1 + \frac{Bs^2}{AE-BC} \right) G_s(s)}{1 - \frac{A}{C} s - \frac{s^2}{C} + G_s(s) K_2 G_r(s) \left(\frac{AE-BC}{C} \right) \left(1 + \frac{E}{AE-BC} s \right)}$$

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Assuming that $G_2(s)$ includes the integration term, the open loop transfer functions for type D will be the same as for type C if K_2 is replaced by $K_2 + K_3 s$.

The information that can be obtained from examination of these open loop transfer functions are the following.

For Case (1).

Effect of servo response and variation thereof. Nonlinearity can be considered by use of Johnson's describing functions.

For Case (2).

Effect of rate gyro response and variation thereof. Effect of additional filter following the gyro.

For Case (3).

Effect of accelerometer response and variation thereof. Effect of additional filter following the accelerometer.

For purposes of seeing how the Nyquist plots look qualitatively, the instrument and servo response can be made ideal.

$$\begin{aligned} G_r(s) &= 1 \\ G_a(s) &= 1 \\ G_s(s) &= 1 \quad \text{For Type C} \\ &= 1/s \quad \text{For Type D} \end{aligned}$$

The following approximations can be made for the aerodynamic coefficients.

$$\begin{aligned} \frac{B}{AE-BC} &\approx 0 \\ AE-BC &\approx AE \end{aligned}$$

The open loop transfer functions will then be.

For the type C system.

$$\begin{aligned} 2.3-4 \quad \frac{\theta_o}{\theta_{in}} &= -\frac{AE}{C} \left[\frac{K_2 + \frac{K_2}{s} + K_1 \frac{V}{1845}}{1 - \frac{A}{C}s - \frac{s^2}{C}} \right] \\ 2.3-5 \quad \frac{\dot{\psi}_o}{\dot{\psi}_{in}} &= -K_2 \left(\frac{AE}{C} \right) \frac{(1 + \frac{s}{A})}{1 - \frac{A}{C}s - \frac{s^2}{C} + \frac{V}{1845} \frac{AE}{C} K_1} \\ 2.3-6 \quad \frac{\ddot{\eta}_o}{\ddot{\eta}_{in}} &= \frac{-K_1 \left(\frac{V}{1845} \right) \frac{AE}{C}}{1 - \frac{A}{C}s - \frac{s^2}{C} + K_2 \left(\frac{AE}{C} \right) (1 + \frac{s}{A})} \end{aligned}$$

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For the type 0 system.

$$2.3-7 \quad \frac{z_0}{z_{in}} = -\frac{AE}{C} \left[\frac{(K_2 + K_3 s)(1 + \frac{s}{A}) + K_1 \frac{V}{1845}}{(1 - \frac{A}{C} s - \frac{s^2}{C}) s} \right]$$

$$2.3-8 \quad \frac{\dot{z}_0}{\dot{z}_{in}} = - (K_2 + K_3 s) \left(\frac{AE}{C} \right) \frac{(1 + \frac{s}{A})}{(s - \frac{A}{C} s^2 - \frac{s^3}{C} + \frac{V}{1845} \frac{AE}{C} K_1)}$$

$$2.3-9 \quad \frac{n_0}{n_{in}} = \frac{-K_1 \left(\frac{V}{1845} \right) \frac{AE}{C}}{s - \frac{A}{C} s^2 - \frac{s^3}{C} + (K_2 + K_3 s)(1 + \frac{s}{A}) \frac{AE}{C}}$$

Since negative feedback was not implied in the derivation of the open loop transfer functions, the characteristic equation is of the form

$$1 - \frac{z_0}{z_{in}} = 0$$

This would mean that the encirclement of +1 on the polar plot should be examined. However, since it is more conventional to examine the encirclement of -1, all the open loop functions should be multiplied by -1 before being plotted. The characteristic equation will then be of the form $1 + (-\frac{z_0}{z_{in}}) = 1 + G(s) = 0$

The simplified form of these open loop expressions are, in general, of the form.

$$-\frac{z_0}{z_{in}} = K \frac{1 + C_1 s + C_2 s^2}{1 + C_3 s + C_4 s^2 + C_5 s^3}$$

and the Nyquist plot can be computed with a little effort. The cases where the aerodynamic coefficient C is positive, there will be poles in the right hand plane and it would be necessary to examine the counterclockwise encirclements of -1 after the number of these poles is determined.

These Nyquists of the simplified open loop transfer function are extremely useful in getting a qualitative understanding of the system. Figure 6 shows a typical example.

2.4. Determination Of Gains.

The actual determination of the gains will in general be an iteration process. This is necessary because all the possible factors that must be considered cannot be included in any one analysis or simulation.

For the Tartar missile the factors that had to be considered were the following:

- (1). Elastic body coupling.

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- (2). Radome error slope coupling.
- (3). Compatibility with roll system in the presence of aerodynamic coupling derivatives.
- (4). Control surface servo limitation.
- (5). Instrument limitations.
- (6). Tolerances.
- (7). Effect of noise.

The desired set of gains would be that which gives the fastest response for after all these factors have been considered.

These factors will be explained in detail.

2.4.1 Elastic Body Coupling.

Figure 4 shows a block diagram of the type D autopilot system with the additional elastic body loop included. The loop coupled through the rigid body aerodynamic responses which had been the main loop considered up to now can be assumed open. This is because the elastic body resonance frequencies are on the order of 350 rad/sec and the rigid body aerodynamic responses cut off at around 10 rad/sec.

The transfer function for the elastic body responses are derived in detail in Appendix 1.

In particular the transfer functions are derived for the case where the solution to the elastic beam partial differential equation is represented only by the first term in the series solution and the body load assumed negligible.

The terms in the series solution represent the various vibration modes of the beam and only the mode with the lowest frequency is considered. This is reasonable since the second mode is at sufficiently high frequency so that the filtering required for the first mode will adequately take care of the second and higher modes. The aerodynamics loads due to local angles of attack were found to have negligible effect on the mode shapes or frequency for a representative flight condition. This means that the solutions for the Tartar bending modes computed for vacuum conditions is approximately the same as for moving air conditions.

The transfer functions for the instrument outputs for an arbitrary force input at the control surface station were found in Appendix 1 to be

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$$2.4-1 \quad \frac{F_T}{F_T} = \frac{\Delta^2 \phi(x_c) \phi(x_a)}{(\Delta^2 + 2.5 \omega_B \Delta + \omega_B^2) (32.2)(12) M_g}$$

$$2.4-2 \quad \frac{\psi_B}{F_T} = \frac{\Delta \phi(x_c) \left(\frac{d\phi}{dx} \right)_{x=x_a} 57.3}{(\Delta^2 + 2.5 \omega_B \Delta + \omega_B^2) M_g}$$

where

$\phi(x)$ = shape of first bending mode computed with an arbitrary displacement of 1 in at station x_c .

$\phi(x_c)$ = displacement at $x = x_c$, the station for the control surface hinge line (in).

$\phi(x_a)$ = displacement at $x = x_a$, the station at which the accelerometer is located (in).

$\left(\frac{d\phi}{dx} \right)_{x=x_a}$ = local slope at $x = x_a$, the station at which the rate gyro is located (rad).

M_g = generalized mass defined by $M_g = \sum_{i=1}^n m_i \phi^2(x_c)$ [where m_i = mass at station x_i and $\phi(x_i)$ = displacement at station i] (lb in sec²).

ω_B = frequency of first bending mode (rad/sec).

η_B = acceleration due to body bending at the accelerometer station (g's).

ψ_B = rate of body bending at the rate gyro station (deg/sec).

F_T = force input at the control surface hinge line (lbs).

ξ = structural damping coefficient.

The transfer function for the elastic body system with the loop opened at the wing for the type D system is

$$2.4-3 \quad \frac{z_a}{F_T} = G_B(s) \left(\left[\left(\frac{\psi_B}{F_T} \right) G_R(s) (K_2 + K_3 s) + \left(\frac{\eta_B}{F_T} \right) G_A(s) K_1 \right] \right)$$

In order to insure that there will be no instability in this loop z_a/z_{in} should be $\zeta = .3$ when $\omega = \omega_B$ for an assumed structural damping of $\omega = .02$. This margin would be insurance against the tolerances on ξ , ω_B , $\phi(x)$, autopilot gains, instrument responses, and components.

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The open loop transfer function needs to be examined only at $\omega = \omega_B$ since at any other ω the amplitude will be less.

Substituting $j\omega_B$ for s into equation 2.4-3

$$2.4-4 \quad \left(\frac{z_e}{z_{in}} \right)_{max} = G(s)(j\omega_B) \left(\frac{F_e}{t} \right) \left[\frac{\phi(x_B) \left(\frac{d\phi}{dx} \right)_{x=x_B} 573 G_x(j\omega_B) (k_2 + k_3 j\omega_B)}{2.5 \omega_B M} + \frac{\phi(x_e) \phi(x_n) G_{ax}(j\omega_B) K_1}{2.5 M (32.2)(7/2)} \right]$$

The obvious method for making $\left(\frac{z_e}{z_{in}} \right)_{max} < .3$ is to locate the instruments such that $\left(\frac{d\phi}{dx} \right)_{x=x_B} = 0$ and $\phi(x_n) = 0$. Figure 5 shows the computed first bending mode shape for the Tartar missile. Due to packaging requirements, the instruments for the Tartar missile are at the following stations:

Accelerometer Station 55
Rate Gyro Station 80

With the instruments at these stations the values for the displacements and slope are

$$\phi(x_B) = .2$$

$$\left(\frac{d\phi}{dx} \right)_{x=x_n} = .01$$

At the control surface hinge line $\phi(x_e) = .5$

The values for the remaining factors are:

$$\omega_B = 345 \text{ rad/sec}$$

$$M = .539 \text{ lb-in-sec}^2$$

$$K_1 = .02$$

Substituting these values into equation 2.4-4

$$\left(\frac{z_e}{z_{in}} \right)_{max} = G_x(j\omega_B) \left(\frac{F_e}{t} \right) \left[\frac{\phi(x_B) \left(\frac{d\phi}{dx} \right)_{x=x_B} 573 G_x(j\omega_B) (k_2 + k_3 j\omega_B)}{2.5 \omega_B M} + \frac{\phi(x_e) \phi(x_n) G_{ax}(j\omega_B) K_1}{2.5 M (32.2)(7/2)} \right]$$

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The terms that will give the greatest contribution is the term with K_3 . Since $K_1 \approx 10K_2 \approx 100K_3$ for the Tartar missile. Neglecting all

but the K_3 term

$$\left(\frac{i_a}{i_{in}}\right)_{max} = G_a(j\omega_B) \left(\frac{F_T}{T}\right) (.0384) G_n(j\omega_B) jK_3 \quad (345)$$

In order to determine the amount of attenuation that is required from $G_a(j\omega_B) \times G_n(j\omega_B) \times$ (filter if required), these can be lumped together into

$$\frac{G_F(j\omega_B)}{j\omega_B} = G_a(j\omega_B) G_n(j\omega_B) G_f(j\omega_B)$$

since $G_a(j\omega) \approx \frac{1}{j\omega}$ for type D.

$$2.4-4 \left(\frac{i_a}{i_{in}}\right)_{max} \approx G_F(j\omega_B) (.0384) K_3 \left(\frac{F_T}{T}\right)$$

$\frac{F_T}{T}$ can be computed from

$$F_T/T = 1481 \lambda A M^2 C_{p_n} \quad (M = \text{Mach Number})$$

The value used for C_{p_n} should be that computed from oscillatory aerodynamics for a frequency of ω_B . For the Tartar missile, however, the values computed from oscillatory aerodynamics did not differ appreciably from that for the stationary aerodynamics.

Studies have shown that the M1.5 condition gives the largest $\left(\frac{i_a}{i_{in}}\right)_{max}$ value for the Tartar missile. This is assuming that K_3 is programed to vary with total pressure.

A sample calculation will be made for the M1.5 S.L. condition

$$\begin{aligned} \frac{F_T}{T} &= (1481) (.994) (1.5)^2 (1.25) \\ &= 434 \end{aligned}$$

$$\left(\frac{i_a}{i_{in}}\right)_{max} \approx G_F(j\omega_B) 15.9K_3$$

The K_3 ultimately arrived at for the Tartar missile for this condition was $K_3 = .275$ (this is the value for K_3 that takes into account the small amount of ψ^3 that the accelerometer senses due to its being off c.g.).

$$\left(\frac{i_a}{i_{in}}\right)_{max} = G_F(j\omega_B) 2.78$$

$$\text{if } \left(\frac{i_a}{i_{in}}\right)_{max} = .3$$

$$G_F(j\omega_B) = \frac{.3}{2.78}$$

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This means that at $\omega = 345$ rad/sec the combination of gyro response, servo response and filter response must give an attenuation of $1/10.8$ or -20.6 D.B. By servo response is meant the response over and beyond that of a perfect integrator. The rate gyro is a second order system and the state of the art is such that its natural frequency and damping ratio can be specified to any arbitrary values within reason. For the Tartar missile the specified nominal values for the rate gyro are

natural frequency = 27 cps

damping ratio = .5

This will provide an attenuation of -12DB at 345 rad/sec. The additional -8 DB can be obtained either from the servo or an additional first order filter. What must be considered in the process of determining how to obtain the desired attenuation is the phase lags that will be introduced at the gain cross over frequencies for the Nyquists of the system.

For the Tartar autopilot system the gain cross over frequency for the Nyquist with the loop opened at either the control servo output or at the rate gyro is about 40 rad/sec. This can be seen by plotting the simplified open loop transfer functions or by making the assumption that at the frequencies concerned

$$\frac{e_p}{e_{in}} \approx \frac{E K_3}{s}$$

and therefore the gain crossover is at $\omega \approx EK_3$. The simplified Nyquist for $1/10.8$, M1.5 S.L. is shown in Figure 6.

At $\omega = 40$ rad/sec the additional phase lag introduced by the rate gyro and the required first order filter are

- (1). From the rate gyro 14°
- (2). From the filter 14°
(corner at 24 cps)

The system must be able to tolerate this additional phase lag and still have an adequate phase margin of $\approx 30^\circ$.

If it turns out that this is not the case, the possible fixes are:

- (1). Move the instruments.
- (2). Design a complex filter that gives the required attenuation at but less lag at system gain crossover frequency.
- (3). Find some means to reduce K_3 .

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For the Tartar missile the additional 28° of phase lag at $\omega = 40$ rad/sec was tolerable.

There exists therefore a choice of putting the first order filter after the instruments or incorporating it in the transfer function of the control servo. If the filter is placed at the output of the instruments, the servo must be designed to introduce as small an amount of phase lag as possible over the nominal 90° at 40 rad/sec. Since this would add to the 28° already introduced. The additional amount that Tartar can tolerate is on the order of 5° . The disadvantage of this is that the requirement on the servo is prohibitively stringent. The advantage is that if the requirement can be met, it is possible to use high gains in the roll system without complex networks. This comes about because the roll system uses the same control surfaces as the autopilot and phase lags in the servo are introduced in the roll system also.

The advantage in incorporating the filter in the servo response is that the requirements on the servo are deliberately relaxed.

Both methods are being considered for the Tartar missile.

2.4.2 Radome Error Slope Coupling.

The equations which show the existence of the coupling due to radome error slope will be derived first. The angles required for the analysis are shown in Figure 7.

These angles are:

σ = Angle between line of sight to target and reference.

θ = Angle between seeker centerline and reference.

ψ = Angle between missile centerline and reference.

β = Angle between seeker centerline and missile centerline.

ϵ = Angle between line of sight to target and centerline of the seeker.

ϵ_1 = Error angle due to radome refraction

$$\beta + \epsilon = \text{Look angle} = (\sigma - \psi).$$

The relationship between these angles are:

$$\epsilon = \sigma - \theta = \sigma - \psi - \beta$$

2.4.5

$$\theta = \beta + \psi$$

$$\epsilon' = \epsilon + \epsilon_1$$

ϵ_1 will in general be a function of the look angle $\beta + \epsilon$

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$$\varepsilon_1 = f(\beta + \varepsilon)$$

and

$$d\varepsilon_1 = \frac{d\varepsilon_1}{d(\beta + \varepsilon)} d(\beta + \varepsilon)$$

if $\frac{d\varepsilon_1}{d(\beta + \varepsilon)}$ is assumed constant and set equal to n

2.4-6 $\varepsilon_1 = n(\beta + \varepsilon)$. Over small regions of look angle this approximation can be made.

Since the dynamics of the seeker is not required for the analysis in this section, it will not be presented. It is sufficient to state that the function of the seeker is to continuously track the target and in the process of doing this provide information regarding the rate of rotation of the line of sight angle to the target ($\dot{\theta}$). This information is immediately available since if the seeker tracks the target with a reasonable degree of accuracy, the rate of rotation of its centerline in space is identically the rate of rotation of the line of sight.

The homing guidance equation is ideally

$$\dot{\theta}_c = \Lambda \frac{V_R}{V_M} \dot{\theta} - G_g(s)$$

where Λ is a constant ≈ 4 and V_R is the relative velocity between target and missile. $G_g(s)$ is the transfer function of the guidance filter.

For the Tartar missile $\dot{\theta}$ information is obtained from $\dot{\theta} + \dot{\varepsilon}'$ or the head rate gyro signal plus the rate of the tracking error signal.

From equations 2.4-5 and 2.4-6

$$\begin{aligned}\dot{\theta} + \dot{\varepsilon}' &= \dot{\theta} + \dot{\varepsilon} + n(\beta + \varepsilon) \\ &= (1+n)\dot{\theta} - n\dot{\theta}\end{aligned}$$

and since

$$n_c(\dot{\theta}) = \frac{V_M}{1845} \dot{\theta}_c \text{ (deg/sec)}$$

$$n_c = \frac{\Lambda V_R}{1845} [(1+n)\dot{\theta} - n\dot{\theta}] G_g(s)$$

For the Tartar missile

$$G_g(s) = \frac{1}{(1 + .25s)(1 + .25s)}$$

This was determined by a study on the amount of filtering required for adequate homing guidance.

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The complete equation for N_c is therefore

$$N_c = \frac{-A V_R}{1845} \left[\frac{(1+\mu) \dot{\psi} - \mu \dot{\psi}}{(1+25\mu)(1+25\mu)} \right]$$

This shows that there is an additional $\dot{\psi}$ feedback into the autopilot. A block diagram of this additional coupling loop is shown in Figure 8.

The open loop transfer function with the loop opened at $\dot{\psi}$ is

$$\frac{\dot{\psi}_o}{\dot{\psi}_i} = \frac{-\mu A V_R}{1845} [G_{\dot{\psi}}(s)] \left[\frac{n}{n_c} \right] \left(\frac{\dot{\psi}}{n} \right)$$

The simplified open loop transfer function for n/n_c is of the form:

$$n/n_c = \frac{1}{(1 + T_n s)(1 + 25 T_n s + T_n^2 s^2)}$$

$\dot{\psi}/n$ is of the form

$$\frac{\dot{\psi}}{n} = \frac{1845}{V_R} (1 + T_{\dot{\psi}} s) \quad (\text{From section 2.1})$$

where $T_{\dot{\psi}} = \frac{E}{AE - BC}$

The open loop transfer function becomes

$$\frac{\dot{\psi}_o}{\dot{\psi}_i} = \frac{(-\mu) A V_R (1 + T_{\dot{\psi}} s)}{V_R (1 + 25\mu)(1 + 25\mu s)(1 + T_n s)(1 + 25 T_n s + T_n^2 s^2)}$$

When μ is positive there is negative feedback in the loop and when it is negative, positive feedback. Since with positive feedback this loop becomes extremely difficult to stabilize, μ can be biased so that in effect it has only positive values. This is done by adding to $\dot{\psi}$ some pitch rate gyro signal $\mu_{max} \dot{\psi}'$.

then

$$\frac{\dot{\psi}_o}{\dot{\psi}_i} = \frac{(\mu_{max} + \mu) A V_R (1 + T_{\dot{\psi}} s)}{V_R (1 + 25\mu)(1 + 25\mu s)}$$

The value of $(\mu_{max} + \mu)$ will never be negative.

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For an $n_{max} = +.04$, the quantity $(\frac{n_{max} + A}{V_M}) V_R A \approx .5$ while T_d , which equals $\frac{F}{AE - GC}$, can be as large as 4.0 at high altitude

conditions where A decreases. The autopilot time constants T_A and T_d must be adjusted to stabilize the loop for these conditions. At low altitude conditions, there is no stability problem in this loop since T_d is small and the attenuation from the guidance filters is more than sufficient. Figure 9 shows a Bode plot of the V_R/V_M open loop transfer function for the M2.0 h = 50,000 ft. flight condition. For a fixed flight condition T_d varies with ω but T_A varies roughly in the same manner thus maintaining the same degree of stability.

An alternate method for stabilizing this loop is the following: Design the auto pilot for maximum possible speed and design the guidance filter to stabilize the loop. The advantages and disadvantages of both methods are discussed in the section on guidance computer.

2.4.3 Compatibility With Roll System In The Presence Of Aerodynamic Coupling Derivatives.

This subject is covered in full in the section on roll system. It is sufficient to say at this point that the gains must not be determined without consideration of the roll yaw coupling problem. In general the roll system alone cannot be designed to stabilize extreme roll-yaw coupling instabilities. Slight modifications of the autopilot gains can improve the situation considerably. As an example, low damping ratios ζ in the pair of complex roots of the autopilot is detrimental in the roll yaw coupling loop. This low ζ can result from the fact that the stability derivatives for yaw motion is different from that for pitch alone when the missile is pitched at a large angle of attack.

2.4.4 Control Surface Servo Limitations.

The following limitations must be considered.

- (1) Maximum practical servo response.
- (2) Maximum wing rate.
- (3) Effect of wing loads (hinge moment) on (1) and (2).
- (b) Non Linear phase lags.

It has already been pointed out in section 2.4.1 that the consideration of the maximum practical servo response determines the method by which body bending mode coupling loop is stabilized.

The effect of maximum wing rate on the stability of the system can be determined by use of the describing function on the Nyquist of the open loop transfer function for the loop opened at the wing servo ($1/s_n$). This will immediately show

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that the type C system with the position feedback servo is more susceptible to non linear oscillation than the type D system with the rate servo. A possible means of improving the situation is to add another non linearity, such as a frequency sensitive limit on the input to the servo.

The effect of wing loads is to reduce the servo speed of response and possibly reduce the maximum wing rate. This must be considered as an extreme tolerance on the servo response.

The non linear phase lags from dead space, sticky value, etc., should be estimated and considered in the ($\frac{1}{\omega} \frac{d\phi}{dt}$) Nyquist plot.

The control surface servo transfer functions are derived in section 3.5.2 This is the section on servo limitations that affect the roll system. The derivation is in this section since the servo response is more critical for the roll system.

2.4.5 Instrument Limitations.

The following factors must be considered.

- (1) Compatibility of specified dynamic response with the dynamic range required.
- (2) Compatibility of the dynamic range required with the null, noise, resolution, g sensitivity and linearity characteristics.

The desired dynamic response can be determined by the method suggested in section 2.4.1 if the response is to be used to stabilize the body bending mode coupling loop. If it is physically possible, this is desirable since the phase lag from the instrument need not be designed out of the system.

The effect of null unbalance, noise, and g sensitivity, can be computed by assuming that these are the only inputs to the system and computing what the output (N_z) will be with these inputs.

As an example the effect of additional inputs at the rate gyro on steady state conditions will be computed.

The control equation is

$$A \cdot \ddot{\alpha} = K_1 (\alpha - \alpha_c) + (K_2 + K_3 \alpha) (\dot{\psi} + \dot{\psi}_0' - \frac{1}{T} \alpha)$$

where $\dot{\psi}_0'$ can be the null value, or the amount due to g sensitivity (assuming max g)

For steady state conditions, $\ddot{\alpha} = 0$

$$K_1 (\alpha - \alpha_c) = -K_2 \dot{\psi}_0'$$

$$\alpha - \alpha_c = -\frac{K_2}{K_1} \dot{\psi}_0'$$

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This gives the amount of error introduced by the rate gyro in the steady state condition.

2.4.6 Tolerance.

The gains must be chosen to be able to stand whatever tolerance can be practically met. For the Tartar, the gains were chosen to give adequate performance with 20% variation in any combination. This was based on the expected accuracy of the P_t meas ring and gain setting device and the component tolerances.

2.4.7 Effect Of Noise.

The effect of noise from any possible source can be computed by assuming one source at a time as the only input. The output considered should be both $(\dot{\gamma}_t)$ and $\dot{\gamma}_t$.

As an example, if there is noise at the accelerometer and it is desired to find its effect on $\dot{\gamma}_t$, the transfer function between $\dot{\gamma}_t$ and A_t must be derived. The control equation with a noise input at the accelerometer is

$$\dot{\gamma}_t = G_F(s) K_1 [n_{acc} + n'] + (K_2 + K_3 s) \psi'$$

where $G_F(s)$ is the filter following the accelerometer.

Since

$$n' = \left(\frac{n}{\gamma_t}\right) G_s(s) G_a(s) \dot{\gamma}_t$$

$$\psi' = \left(\frac{\psi}{\gamma_t}\right) G_s(s) G_a(s) \dot{\gamma}_t$$

($G_a(s)$ contains $\frac{1}{s}$)

$$\dot{\gamma}_t = G_F(s) K_1 n_{noise} + G_F(s) K_1 G_a(s) G_s(s) \left(\frac{n}{\gamma_t}\right) \dot{\gamma}_t + (K_2 + K_3 s) G_a(s) G_s(s) \left(\frac{\psi}{\gamma_t}\right) \dot{\gamma}_t$$

$$\frac{\dot{\gamma}_t}{n_{noise}} = \frac{G_F(s) K_1}{1 - G_F(s) K_1 G_a(s) G_s(s) \left(\frac{n}{\gamma_t}\right) - (K_2 + K_3 s) G_a(s) G_s(s) \left(\frac{\psi}{\gamma_t}\right)}$$

$$= G(s)$$

The RMS $\dot{\gamma}_t$ can be computed from

$$(\dot{\gamma}_{t_{RMS}})^2 = \frac{I_n}{2\pi^2} \int_{-\infty}^{+\infty} G^2(s) \omega^2 ds$$

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where E_o is the spectral density of the accelerometer noise in $g^2/\text{rad}/\text{sec}$.

Normally it is sufficient to throw out all the terms in the denominator of $\frac{1}{1+G(S)}$ except the 1 and $G(S)$ becomes

noise $G(\Delta) \approx G_F(\Delta) K,$

This is justified for two reasons (1) the feedback terms are attenuated quite heavily and (2) the RMS calculation is more conservative without them.

The $\dot{\phi}_{c, \text{RMS}}$ can now be simply computed.

3. ROLL SYSTEM DYNAMICS

3.1 Description of System

Appendix II describes the system that is to be analyzed. This system has the following degrees of freedom:

- (1) y = translation in the yaw plane.
- (2) ψ = rotation about the c.g. in the yaw plane.
- (3) ϕ = rotation about the missile centerline.
- (4) δ_y = control surface deflection to produce yaw force and moment.
- (5) δ = control surface deflection to produce rolling moment.

The angles and the stability derivatives required are all defined in Appendix II.

3.2 Form of Control Equation.

The type D control system will be assumed for the autopilot.

The roll system control equation can again have any number of forms. The form that the Tartar missile uses is the following

3.2-1 $\delta_c = - \frac{(K_d + K_p S + K_i S^2)}{S} \phi'$

When the equation is actually implemented it takes the form

3.2-1' $\dot{\delta}_c = -K_d(\phi' + K_p \phi'')(1 + T S)$

Figure 10 shows a block diagram of the system. $G_\phi(S)$ is the free gyro response, $G_{\dot{\phi}}(S)$, the rate gyro response and ϕ' and ϕ'' the outputs of the two instruments.

3.3 Simplified Transfer Functions

The input to the roll system can be either a command to roll or an extraneous

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roll torque ($\ddot{\phi}_c$). The output or the controlled variable is ϕ . The simplified closed loop transfer functions with these two inputs can be derived for the system by assuming that the roll-yaw coupling derivatives are all zero, the instruments have perfect responses, and the servo is a perfect integrator.

The closed loop transfer function with a called for roll angle input will be derived first.

Using the form of the control equation 3.2-1'

$$\dot{\delta} = K_a(1+Ts)(\phi_c - \phi) - K_D K_a(1+Ts)\dot{\phi}$$

Where $(\phi - \phi_c)$ is the free gyro output.

$$\dot{\delta} = -[K_a + (KT + K_D K_a)s + K_D K_a T s^2]\phi + K(1+Ts)\phi_c$$

The roll moment equation is

$$\ddot{\phi} = F\dot{\phi} + G\delta$$

and

$$\frac{\phi}{\delta} = \frac{G}{s(s-F)}$$

Substituting into the control equation

$$\left[\frac{s^2(s-F)}{G} + K_a + (K_D T + K_D K_a)s + K_D K_a T s^2 \right] \phi = K_a(1+Ts)\phi_c$$

and therefore

$$3.3-1 \quad \frac{\phi}{\phi_c} = \frac{1+Ts}{\frac{s^3}{GK_a} + (K_D T - \frac{F}{GK_a})s^2 + (T + K_D)s + 1}$$

For a step $\ddot{\phi}_c$ the roll moment equation is

$$\ddot{\phi} = G\delta + F\dot{\phi} + \ddot{\phi}_c$$

or

$$(s^2 - sF)\phi - G\delta = \ddot{\phi}_c$$

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with the control equation

$$[K_a + (K_d T + K_D K_a) s + K_D K_a T s^2] \phi + s \delta = 0$$

$$\frac{\phi}{\phi_c} = \frac{s}{s^3 - s^2 F + G K_a + G(K_d T + K_D K_a) s + G K_D K_a T s^2}$$

and therefore

$$3.3-2 \quad \frac{\phi}{\phi_c} = \frac{\frac{s}{G K_a}}{\frac{s^3}{G K_a} + (K_d T - \frac{F}{G K_a}) s^2 + (T + K_D) s + 1}$$

Equations 3.3-1 and 3.3-2 give the basic characteristics of the system. The roll system in the absence of roll-yaw coupling is a third order system. The static gain for a step ϕ_c is identically unity. With a step ϕ_c the system behaves like an imperfect differentiator.

There are again three possible places to open the loop.

The open loop transfer function for the loop opened at the servo is

$$3.3-3 \quad \frac{\delta_o}{\delta_{in}} = \frac{(K_a + K_5 s + K_6 s^2) G}{s^2 (s - F)}$$

3.4 Analysis of The Complete System

The equations of motion for the five degrees of freedom are (from Appendix II),

$$A\beta + B\dot{\gamma} + M\dot{\delta} = \beta - \dot{\phi} \sin \alpha_0 + \dot{\gamma} \cos \alpha_0$$

$$C\beta + E\dot{\gamma} + N\dot{\delta} = \dot{\gamma}$$

$$\dot{\gamma} = K_1'(A\beta + B\dot{\gamma} + M\dot{\delta}) + K_2 \dot{\gamma} + K_3 \dot{\gamma}$$

$$G\dot{\delta} + F\dot{\phi} + H\beta + L\dot{\gamma} = \dot{\phi}$$

$$\dot{\delta}_c = -(K_a + K_5 s + K_6 s^2) \phi$$

where

$$K_1' = \frac{Y}{1845} K_1$$

rearranging

$$(A-s)\beta + B\dot{\gamma} - \cos \alpha_0 \dot{\gamma} + \sin \alpha_0 \dot{\phi} + M\dot{\delta} = 0$$

$$C\beta + E\dot{\gamma} - s\dot{\gamma} + N\dot{\delta} = 0$$

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(equation continued) $K_1 A \beta + (K_1 B - S) l_y + (K_2 + K_3 S) \dot{\psi} + MK_1 \delta = 0$
 $-H \beta - L l_y + (S - F) \dot{\phi} - G \delta = 0$
 $(K_4 + K_5 S + K_6 S^2) \dot{\phi} + S^2 \delta = 0$

The term $\frac{I_y - I_x}{I_z} \frac{\dot{\phi} \dot{\psi}}{57.3}$ is omitted from these equations. Since for the Tartar it was found to have negligible effect.

these equations are of the form

$$\begin{aligned} A_{11} \beta + A_{12} l_y + A_{13} \dot{\psi} &= -\sin \alpha_0 \dot{\phi} - M \delta \\ A_{21} \beta + A_{22} l_y + A_{23} \dot{\psi} &= -N \delta \\ A_{31} \beta + A_{32} l_y + A_{33} \dot{\psi} &= -MK_1 \delta \\ H \beta + L l_y &= A_{44} \dot{\phi} + A_{45} \delta \\ 0 &= A_{54} \dot{\phi} + A_{55} \delta \end{aligned}$$

where

$$A_{11} = A - S$$

$$A_{12} = B$$

$$A_{13} = -\cos \alpha_0$$

$$A_{21} = C$$

$$A_{22} = E$$

$$A_{23} = -S$$

$$A_{31} = K_1 A$$

$$A_{32} = K_1 B - S$$

$$A_{33} = K_2 + K_3 S$$

$$A_{44} = S - F$$

$$A_{45} = -G$$

$$A_{54} = K_4 + K_5 S + K_6 S^2$$

$$A_{55} = S^2$$

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defining the determinants

$$\begin{vmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{vmatrix} \triangleq \Delta_{13}$$
$$\begin{vmatrix} A_{44} & A_{45} \\ A_{54} & A_{55} \end{vmatrix} \triangleq \Delta_{45}$$

and
3.4-1 $I \triangleq H_{\beta} + I_{\beta}$

$$\phi = \frac{I A_{55}}{\Delta_{45}}$$

3.4-2

$$\delta = \frac{-I A_{54}}{\Delta_{45}}$$

3.4-3

$$\beta = \frac{-\sin \alpha_0 \begin{vmatrix} A_{22} & A_{23} \\ A_{32} & A_{33} \end{vmatrix} \dot{\phi} - \begin{vmatrix} M & A_{12} & A_{13} \\ N & A_{22} & A_{23} \\ MK_1 & A_{32} & A_{33} \end{vmatrix} \delta}{\Delta_{13}}$$

3.4-4

$$\dot{\beta} = \frac{\sin \alpha_0 \begin{vmatrix} A_{21} & A_{23} \\ A_{31} & A_{33} \end{vmatrix} \dot{\phi} + \begin{vmatrix} M & A_{11} & A_{13} \\ N & A_{21} & A_{23} \\ MK_1 & A_{31} & A_{33} \end{vmatrix} \delta}{\Delta_{13}}$$

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Multiplying 3.4-3 by H and 3.4-4 by L and adding

$$H\beta + L\gamma = I = \left\{ -H \begin{vmatrix} A_{22} & A_{23} \\ A_{32} & A_{33} \end{vmatrix} + L \begin{vmatrix} A_{21} & A_{23} \\ A_{31} & A_{33} \end{vmatrix} \right\} \sin \phi_0 \dot{\phi}$$

Substituting equations 3.4-1 and 3.4-2

$$+ \left\{ -H \begin{vmatrix} M & A_{12} & A_{13} \\ N & A_{22} & A_{23} \\ MK_1 & A_{32} & A_{33} \end{vmatrix} + L \begin{vmatrix} M & A_{11} & A_{13} \\ N & A_{21} & A_{23} \\ MK_1 & A_{31} & A_{33} \end{vmatrix} \right\} \Delta_{13}$$

3.4-5

$$\frac{I}{I} = \frac{\left\{ -H \begin{vmatrix} A_{22} & A_{23} \\ A_{32} & A_{33} \end{vmatrix} + L \begin{vmatrix} A_{21} & A_{23} \\ A_{31} & A_{33} \end{vmatrix} \right\} \sin \phi_0 A_{55}}{\Delta_{13} \Delta_{45}}$$

$$+ \left\{ +H \begin{vmatrix} M & A_{12} & A_{13} \\ N & A_{22} & A_{23} \\ MK_1 & A_{32} & A_{33} \end{vmatrix} - L \begin{vmatrix} M & A_{12} & A_{13} \\ N & A_{21} & A_{23} \\ MK_1 & A_{31} & A_{33} \end{vmatrix} \right\} A_{54}$$

$\Delta_{13} \Delta_{45}$

The expression on the right hand side of equation 3.4-5 is the open loop transfer function for the complete system with the loop opened at the input to the roll system. Figure 11 show a block diagram of the complete system.

Equation 3.4-5 can be put in the form

$$\frac{I}{I} = \left\{ \frac{-H \sin \phi_0 \begin{vmatrix} A_{22} & A_{23} \\ A_{32} & A_{33} \end{vmatrix}}{\Delta_{13}} + \frac{L \sin \phi_0 \begin{vmatrix} A_{21} & A_{23} \\ A_{31} & A_{33} \end{vmatrix}}{\Delta_{13}} \right\} \frac{A_{55}}{\Delta_{45}} + \left\{ \frac{+H \begin{vmatrix} M & A_{12} & A_{13} \\ N & A_{22} & A_{23} \\ MK_1 & A_{32} & A_{33} \end{vmatrix}}{\Delta_{13}} - \frac{L \begin{vmatrix} M & A_{12} & A_{13} \\ N & A_{21} & A_{23} \\ MK_1 & A_{31} & A_{33} \end{vmatrix}}{\Delta_{13}} \right\} \frac{A_{54}}{\Delta_{45}}$$

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or 3.4-6

$$\frac{I_o}{I_{ia}} = \left[H \left(\frac{\beta}{\phi} \right) + L \left(\frac{\gamma}{\phi} \right) \right] \left(\frac{\dot{\phi}}{H\beta + L\gamma} \right) + \left[H \left(\frac{\beta}{\delta} \right) + L \left(\frac{\gamma}{\delta} \right) \right] \left(\frac{\dot{\delta}}{H\beta + L\gamma} \right)$$

Substituting the expressions for A_{ij} and expanding the determinants

$$\frac{\beta}{\phi} = \frac{\sin \alpha_0 s^2 - (EK_3 \sin \alpha_0 + BK_1' \sin \alpha_0)s - EK_2 \sin \alpha_0}{\Delta_{13}}$$

$$\frac{\gamma}{\phi} = \frac{(\sin \alpha_0 CK_3 + \sin \alpha_0 K_1' A)s + \sin \alpha_0 CK_1}{\Delta_{13}}$$

$$\frac{\beta}{\delta} = \frac{Ms^2 - (MEK_3 + N \cos \alpha_0 - NBK_3)s - (ME - NB)(K_1' \cos \alpha_0 + K_2)}{\Delta_{15}}$$

$$\frac{\gamma}{\delta} = \frac{(MK_1' + NK_3)s^2 + (NK_2 + MKK_3 - NAK_3)s + (MC - NA)(K_1' \cos \alpha_0 + K_2)}{\Delta_{15}}$$

where

$$\Delta_{13} = s^3 - (A + BK_1' + EK_3)s^2 + [C \cos \alpha_0 - EK_2 + K_3(AE - BC)]s + (AE - BC)(K_1' \cos \alpha_0 + K_2)$$

$$\frac{\phi}{H\beta + L\gamma} = \frac{s^2}{s^3 + (GK_6 - F)s^2 + GK_5s + GK_4}$$

$$\frac{\delta}{H\beta + L\gamma} = \frac{-K_4 - K_5s - K_6s^2}{s^3 + (GK_6 - F)s^2 + GK_5s + GK_4}$$

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If the encirclement of -1 on the Nyquist plot is to be examined $-\frac{I_o}{I_{1n}}$ is plotted since negative feedback is not implied at the point where the loop is opened.

Of the two terms on the right hand side of equation 3.4-6 the first is referred to as the ϕ loop transfer function and the second the δ loop transfer function. Examination of the Nyquist plots for these two loops and the vector sum of the two give an indication of the gain margin existing for the system for variations in H and L together. No information regarding the phase margin in the δ loop alone or the ϕ loop alone can be obtained. In order to obtain these information the expressions for $\frac{I_o}{\delta_{1n}}$ and $\frac{I_o}{I_{1n}}$ must be derived. This can be done in a manner similar to how the expression for $\frac{I_o}{I_{1n}}$ was derived.

Figure 12 shows the Nyquist plots for the ϕ loop, the δ loop and the sum of the two for the Tartar missile. The flight conditions are M 2, 30,000 ft. altitude and $\alpha = 20^\circ$. This was found to be the most critical set of conditions for the Tartar missile. The roll system gains were chosen to attenuate the amplitude in the region of 4 to 40 rad/sec. For more extreme conditions (large α 's higher Mach number), the following happens: The zero frequency amplitude of the δ loop increases and approaches -1. The magnitude of K_δ and K_ϕ required to attenuate the amplitude sufficiently in the 4 - 40 rad/sec region becomes impractical. The zero frequency gain of the δ loop depends only on the aerodynamic coefficients and so if it approaches -1, there is no simple fix.

3.5 Determination of Gains

For a homing missile roll system the requirements on the speed of response ($\frac{\phi}{s}$) or ($\frac{\phi}{\omega_c}$) are not critical since the guidance information does not require the roll attitude of the missile for a reference. This means that the main criteria for the system design is adequate stability. In designing the system for adequate stability the following factors must be considered.

- (1) Body torsional mode coupling
- (2) Control surface servo limitations
- (3) Instrument Limitations
- (4) Tolerance
- (5) Noise

In general it has been found that the method for stabilizing the system is to increase the gains.

The manner in which these factors affect the maximum gains usable will be explained.

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3.5.1 Body Torsional Mode Coupling

The same equations that were used in the bending mode coupling analysis are applicable here except that the torsional mode shape is used and the input is a generalized moment input at the tail.

The rate gyro output can be computed from

$$\dot{\phi}' = \frac{57.3}{s^2 + 2\zeta\omega_T s + \omega_T^2} \frac{\Theta(x_c)\Theta(x_e)}{J'} M_T$$

where J = generalized moment of inertia

$\Theta(x_c)$ = computed angular displacement at the rate gyro station.

$\Theta(x_e)$ = computed angular displacement at the tail

M_T = torque input at the tail

ω_T = torsional natural frequency (first mode)

The same information can be obtained from vibration test data. The test data gives the following information to the STV-6 missile.

975 in lbs at the tail produces a maximum of 154.2 rad/sec² angular acceleration at all stations from 80" forward when the frequency is 142.5 cps.

This says

$$\left(\frac{\text{rad}}{\text{sec}^2}\right) \dot{\phi}' = \left(\frac{154.2}{975}\right) M_T \text{ (in lbs)}$$

or

$$\left(\frac{\circ}{\text{sec}}\right) \dot{\phi}' = \frac{(154.2)(57.3)(12)}{(975)(142.5)(6.28)} M_T \text{ (ft lbs)}$$

$$\frac{\dot{\phi}'}{M_T} = .1213 \left(\frac{\circ/\text{sec}}{\text{ft lbs}}\right)$$

The maximum open loop gain for the torsional mode coupling loop becomes

$$\frac{S_o}{S_{in}} \approx \left(\frac{\delta}{\dot{\phi}'}\right) \left(\frac{\dot{\phi}'}{M_T}\right) \left(\frac{M_T}{\delta}\right) G_f(s)$$

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since $K_6 f \omega_T \gg K_5$ and $G_s(f \omega_T) \approx \frac{1}{f \omega_T}$

and $\frac{\delta}{\psi} = (K_5 + K_6 s) G_s(s) \approx K_6$

$$\frac{M_T}{\delta} = 1481 \lambda s d M^2 C_{\theta_f}$$

for $M 1.5$, $S.L.$, $M^2 C_{\theta_f} = .308$; $K_6 = .013$

$$\frac{M_T}{\delta} = 511 \text{ (lb/ft)}^2$$

$$\left(\frac{\delta_o}{\delta_{in}} \right) = G_r(f \omega_T) (.013) (.1213) (511) = .8 G_F(f \omega_T)$$

This says that at this condition the attenuation required from $G_r(f \omega_T)$ for a value of .3 for $\left(\frac{\delta_o}{\delta_{in}} \right)$ max is

$$G_F(f \omega_T) = \frac{.3}{.8} = .375$$

or -8.5 DB .

A rate gyro with a natural frequency of 85 cps and $\xi = .5$ has been specified for the Tartar missile. This will provide an attenuation of -7.5 DB . The additional -1 DB can be obtained easily from the filter that is normally required after the instrument demodulators.

The torsional mode coupling loop therefore does not present too great a problem for the K_6 that has been picked.

3.5.2 Control Surface Servo Limitations

For the roll system it has been found that torsional mode coupling is not too great a problem. It is possible therefore to have a higher crossover frequency for the loop opened at the servo. From equation 3.3-3

$$\frac{\delta_o}{\delta_{in}} = - \frac{(K_4 + K_5 s + K_6 s^2) G}{s^2(s - F)}$$

The cross over frequency can be roughly estimated by

$$\frac{\delta_o}{\delta_{in}} \approx \frac{G K_6}{s} \quad s \rightarrow \infty$$

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and ω crossover $\approx GK_c$

For the Tartar missile this was set to be ≈ 90 rad/sec. At this frequency the phase lags introduced by the servo becomes extremely significant. It has been mentioned in the section on autopilot design that there is a limitation on the maximum speed of response obtainable.

The type of control surface servo used will be analyzed briefly. Figure 12 shows a block diagram of the servo. The servo amplifier is not an operational amplifier and the input network must be included in the analysis of the system.

The open loop and closed loop transfer functions of the system will be derived using the following notations and assumptions:

Foreward loop gain = $K = K_g K_v K_A$

$$G_v(s) = \frac{1}{1 + 2\zeta T_v s + T_v^2 s^2}$$

$$G_A(s) = 1$$

Source impedance for inputs = 0

Feedback pot impedance = 0

The open loop transfer function with the loop opened at G, the input to the servo amplifier will be derived first

$$K G(s) = \frac{K}{s(1 + 2\zeta T_v s + T_v^2 s^2)}$$

$$H(s) = \frac{\bar{Z}_{in}}{\bar{Z}_{in} + \bar{Z}_R}$$

$$\bar{Z}_{in} = \frac{R/c_1}{R + 1/c_1} = \frac{R}{sRC_1 + 1}$$

$$H(s) = \frac{\frac{R}{sRC_1 + 1}}{\frac{R}{sRC_1 + 1} + \frac{1}{sC_2}} = \frac{sC_2 R}{s^2 R(C_1 + C_2) + 1}$$

$$3.5.2-1 \quad \frac{E_o}{E_{in}} = H(s) K G(s) = \frac{KC_2 R}{(1 + 2\zeta T_v s + T_v^2 s^2)(1 + sR(C_1 + C_2))}$$

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The closed loop transfer function is computed by summing the currents to the grid (G).

$$\frac{e_1}{R} + e_2 s C_1 + e_0 s C_2 = e_g \left[\frac{1}{R} + s(C_1 + C_2) \right]$$

and

$$e_0 = -e_g \frac{K}{(1 + 2sT_v s + T_v^2 s^2)s}$$

substituting for e_g in the first equation

$$\begin{aligned} \frac{e_1}{R} + e_2 s + e_0 s C_2 &= \frac{-1 + 2sT_v s + T_v^2 s^2}{K} \left[\frac{1}{R} + s(C_1 + C_2) \right] \\ e_1 + e_2 s R C_1 &= -e_0 \left[s C_2 R + s(1 + 2sT_v s + T_v^2 s^2) \left(\frac{1}{K} + \frac{s(C_1 + C_2)R}{K} \right) \right] \\ &= -e_0 s \left[C_2 R + \frac{1}{K} + \left(\frac{2sT_v}{K} + \frac{(C_1 + C_2)R}{K} \right) s + \left\{ 2sT_v \frac{(C_1 + C_2)R}{K} \right. \right. \\ &\quad \left. \left. + \frac{T_v^2}{K} \right\} s^2 + T_v^2 \frac{(C_1 + C_2)R}{K} s^3 \right] \end{aligned}$$

3.5.2-2

$$-e_0 = \frac{\frac{e_1}{\frac{1}{K} + C_2 R} + \frac{e_2 s R C_1}{\frac{1}{K} + C_2 R}}{1 + \frac{2sT_v + (C_1 + C_2)R}{K(1 + C_2 R)} s + \frac{2sT_v(C_1 + C_2)R + T_v^2}{K(1 + C_2 R)} s^2 + \frac{T_v^2(C_1 + C_2)R}{K(1 + C_2 R)} s^3}$$

With the actual system the input e_1 is made up of $K_1(\psi - \phi)$, $K_2 \dot{\psi}$ and $K_3(\dot{\phi} + K_0 \phi)$ while the input e_2 is made up of $K_3 \dot{\psi}$ and $K_4(\dot{\phi} + K_0 \phi)$

An examination of equation 3.5.2-1 shows that the gain cannot be increased arbitrarily without increasing $R(C_1 + C_2)$ at the same time resulting in no improvement of the response. In effect the valve natural frequency sets an upper limit on the speed of response that can be attained without compensation. Theoretically the response can be improved by use of a lead network.

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For the Tartar servo a lead network is not used because of hardware considerations. A passive lead network would attenuate the D.C. gain requiring that the gain of the amplifier be increased. This was not practical.

The parameters for the Tartar servo as they now exist are

$$K = 4000$$

$$C_1 = .49$$

$$R = .13$$

$$C_2 = .05$$

With these parameters, the phase lags from the servo over and beyond the ideal 90° are at 40 rad/sec 4° , 50 rad/sec 15° for nominal conditions. These were experimentally determined. With loads on the surfaces they are expected to increase by about 50%.

The Nyquist for the simplified open loop transfer function (δ_o / δ_{in}) for the Tartar roll system shows that there is a phase margin of about $\approx 65^\circ$. If the servo subtracts $\approx 15^\circ$ and the rate gyro response, $\approx 10^\circ$, there remains $\approx 40^\circ$. The filter following the demodulator which is required to remove the 800 cps ripple can subtract another 10° and the remaining phase margin will be 30° .

If, however, the other effects mentioned in the section 2.4.3 contribute more phase lags it would be necessary to use a complex lead-lag network following the demodulator.

The affect of an unbalance in the servo amplifier can be computed by assuming an input exists at the grid

$$\delta_o = - \frac{K}{s} \left[\frac{Z_{in}}{Z_p + Z_{in}} \delta_o + e b \right]$$

where $e b$ is the equivalent input at the grid of the servo amplifier.

then

$$\delta_o \left[1 + \frac{K}{s} \left(\frac{Z_{in}}{Z_p + Z_{in}} \right) \right] = - \frac{K}{s} e b$$

or for steady state condition

$$\delta_o \approx \frac{Z_p + Z_{in}}{Z_{in}} e b$$

$s \rightarrow 0$

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and for the Tartar Servo

$$\int_{0.5\text{ sec}} = \frac{1}{C_2 R} \text{ sec} = \frac{e b}{.0065}$$

The feedback gain therefore determines the effect of the unbalance. The effect of this equivalent \int_{bias} can be then computed for the overall system by substituting it into the control equation.

3.4.3 Instrument Limitations, Tolerance, and Noise

The effect of these factors can be handled in a manner similar to that for the autopilot.

4. GUIDANCE SYSTEM:

The guidance system will be defined as the system with the following input and output.

Input = geometrical line of sight rate

Output = missile acceleration

With this definition the receiver, the seeker dynamics, the autopilot and the guidance computer is included. The receiver will be considered only as an imperfect error sensing device and a source of noise. Figure 13 shows a simplified block diagram of the complete system. The angles and terms used have already been defined in figure 7 and section 2.4.2. The seeker control system and the guidance computer will be discussed briefly.

4.1 Seeker Control System

The seeker control system has two functions (1) to track the target i.e. to keep the dish nutation axis on the target; and (2) to maintain the dish nutation axis in space from rotating when there is no signal from the receiver. The control system shown in the block diagram is that for a DPN-24 type system.

The operation of the system will be explained qualitatively for the following cases: (1) when the line of sight is rotating at a constant rate and (2) when the missile is pitching at a constant rate. Figure 13 must be referred to in order to follow the explanation.

Case (1) when the line of sight is rotating at a constant rate ($\dot{\theta}$), the head must rotate at the same rate to continue tracking. This means that there will be

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a $\dot{\beta}$ equal to $\dot{\sigma}$ and since $\dot{\psi}$ is assumed zero $\dot{\sigma}$ will also equal $\dot{\sigma}$. If β is to be constant however the input to the actuator in the servo must be constant. This means that the input to the servo amplifier which is an integrator must be zero. In order for this to be true ϵ' must equal $\dot{\sigma}$ which in turn equals $\dot{\sigma}$. In other words there will be a tracking error of $\epsilon' = \dot{\sigma}$.

Case (2) when the missile rotates in pitch at a constant rate ($\dot{\psi}$), β will rotate at the same rate but in the opposite direction to drive $\dot{\sigma}$ (which is equal to $\dot{\beta} + \dot{\psi}$) to zero. In the absence of radome error slope this can be accomplished resulting in no tracking error being required. The system will therefore maintain both ϵ' and $\dot{\sigma}$ at zero even though there is a pitch rotation.

4.2 Guidance Computer

4.2.1 Guidance Signal

The input to the guidance computer should be $\dot{\sigma}$. It has been found in the previous section that either ϵ' or $\dot{\sigma}$ is a measure of $\dot{\sigma}$ in the steady state conditions. Dynamically, however, this is not true. The transfer function for ϵ' will be derived first.

$$\epsilon' = \epsilon + r(\sigma - \psi)$$

$$\ddot{\beta}/K = 4\epsilon' - (\dot{\beta} + \dot{\psi})$$

$$\epsilon = \sigma - \psi - \beta$$

$$\text{or } \dot{\beta} + \dot{\psi} = \dot{\sigma} - \dot{\epsilon}$$

$$\ddot{\beta}/K = 4\epsilon' - \dot{\sigma} + \dot{\epsilon}$$

$$\frac{\dot{\sigma} - \dot{\psi} - \dot{\epsilon}}{K} = 4\epsilon' - \dot{\sigma} + \dot{\epsilon}$$

$$\frac{\dot{\sigma} - \dot{\psi} - \dot{\epsilon} + r(\dot{\sigma} - \dot{\psi})}{K} = 4\epsilon' - \dot{\sigma} + \dot{\epsilon} - r(\dot{\sigma} - \dot{\psi})$$

$$\frac{\ddot{\sigma}(1+r)}{K} - \frac{(1+r)\ddot{\psi}}{K} - \ddot{\sigma} + r(\dot{\sigma} - \dot{\psi}) = \frac{\ddot{\epsilon}}{K} + \dot{\epsilon}' + 4\epsilon'$$

$$\text{or } \ddot{\epsilon}' = \frac{\ddot{\sigma}(1+r)}{K} + \frac{\ddot{\sigma}(1+r)}{4K} - \frac{\ddot{\psi}}{4} - \frac{(1+r)\ddot{\psi}}{4K}$$
$$4.2-1 \quad \ddot{\epsilon}' = \frac{\ddot{\sigma}}{4K} + \frac{\ddot{\sigma}}{4} + 1$$

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This shows that if ϵ' were used for guidance σ would be measured along with $\dot{\sigma}$, $\dot{\psi}$ and $\ddot{\psi}$. There will also be a time lag.

The expression for $\dot{\sigma}$ will be derived next

$$\frac{\dot{\epsilon}}{K} = 4\epsilon' - \dot{\sigma}$$

$$\epsilon = \sigma - \dot{\sigma}$$

$$\epsilon' = (\sigma - \dot{\sigma}) + r(\sigma - \psi)$$

$$\dot{\epsilon} = \dot{\sigma} - \ddot{\sigma} - \dot{\psi}$$

$$\frac{\dot{\sigma} - \ddot{\sigma} - \dot{\psi}}{K} = 4(\sigma - \dot{\sigma}) + 4r(\sigma - \psi) - \dot{\sigma}$$

$$\frac{\ddot{\sigma}}{K} + \dot{\sigma} + 4\dot{\sigma} = \frac{\ddot{\psi}}{K} + 4\sigma + 4r\sigma - 4r\psi$$

$$4.2-4 \quad \dot{\sigma} = \frac{\dot{\sigma}(r+1) - r\dot{\psi} + \frac{\ddot{\psi}}{4K}}{\frac{\ddot{\sigma}}{4K} + \frac{5}{4} + 1}$$

This again contains the time lag and a pitch rate coupling term.

If, however, the combination of the signals $\dot{\epsilon}' + \dot{\sigma}$ is used the following results:

Differentiating equation 4.2-1

$$\dot{\epsilon}' = \frac{\sigma(r+1)(1 + \frac{r}{4} + \frac{\ddot{\sigma}}{4K}) - \dot{\sigma}(r+1) - r\dot{\psi}(1 + \frac{r}{4} + \frac{\ddot{\sigma}}{4K}) + r\dot{\psi} - \frac{\ddot{\psi}}{4K}}{1 + \frac{5}{4} + \frac{\ddot{\sigma}}{4K}}$$

$$\text{or} \quad \dot{\epsilon}' = \dot{\sigma}(r+1) - r\dot{\psi} + \frac{-\dot{\sigma}(r+1) + r\dot{\psi} - \frac{\ddot{\psi}}{4K}}{1 + \frac{5}{4} + \frac{\ddot{\sigma}}{4K}}$$

$$4.2-3 \quad \dot{\sigma} + \dot{\epsilon}' = \dot{\sigma}(r+1) - r\dot{\psi}$$

This same result was obtained in section 2.4.2 by considering only the definition of the angles. However to get the effect of reduced $\dot{\epsilon}'$ or $\dot{\sigma}$ the above equations must be used.

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The $\dot{\theta} + \dot{\epsilon}$ signal contains no time lags. The inevitable radome coupling term still exists however.

The $\dot{\psi}/4K$ coupling term is no longer present. In the past when ϵ' was used for guidance the $\dot{\psi}/4K$ term was destabilizing. For the DPM-24 system K was ≈ 70 . Analysis showed that if K decreased to ≈ 50 there will be stability problems.

With the $\dot{\theta} + \dot{\epsilon}$ signal used for guidance however there is no restriction on the value of K during guidance phases of the flight. The maximum value for K can therefore be determined by the requirements on the head response during the nonguided phases of flight.

There is also an advantage in using the $\dot{\theta} + \dot{\epsilon}$ guidance signal from system bias consideration. With this signal only the bias from the rate gyro appears as a guidance signal. All the other biases such as ϵ' bias, and head servo biases result in a constant tracking error and since ϵ' is differentiated the constant tracking error does not appear in the guidance signal.

4.2.2 Effect of Radome on Guidance.

Using $\dot{\theta} + \dot{\epsilon}$ as the guidance signal the closed loop transfer function for n/n_c can be derived assuming a constant n . (The open loop transfer function has already been derived in section 2.4.2).

$$\begin{aligned} n_c &= \frac{\sqrt{1845}}{1845} G_g(s) [\dot{\theta}' + \dot{\epsilon}'] \\ &= \frac{\sqrt{1845}}{1845} G_g(s) [\dot{\theta}(r+1) - r\dot{\psi}] \end{aligned}$$

Since n/n_c is of the form

$$\begin{aligned} n/n_c &= \frac{1}{(1 + T_R s)(1 + 2.5 T_a s + T_a^2 s^2)} & (\text{section 2.2}) \\ \text{and } \dot{\psi}/h &= \frac{1845}{V_M} (1 + T_\psi s) & (\text{Section 2.1}) \end{aligned}$$

where $T_\psi = \frac{E}{AE - BC}$

$$n(1 + T_R s)(1 + 2.5 T_a s + T_a^2 s^2) = \frac{\sqrt{1845}}{1845} G_g(s) \left[\dot{\theta}(r+1) - \left(\frac{1845}{V_M} \right) (1 + T_\psi s) h \right]$$

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Setting

$$G_g(s) = \frac{1}{(1 + T_F s)(1 + T_R s)}$$

$$\Delta \frac{V_E}{V_{A1}} \triangleq N$$

$$\begin{aligned} n \left[(1 + T_F s)^2 (1 + T_A s) (1 + 2 \xi T_a s + T_a^2 s^2) + N r + N r T_{\psi}^1 s \right] \\ = \frac{\Delta V_R}{1845} (r + 1) \dot{\sigma} \end{aligned}$$

or

$$\begin{aligned} \frac{n}{\dot{\sigma}} = & \frac{\Delta V_R}{1845} (r + 1) \\ & 1 + N r + (2 T_F + T_A + 2 \xi T_a + N r T_{\psi}^1) s \\ & + [T_F^2 + 2 T_F T_a + 2 \xi T_a (2 T_F + T_A) + T_a^2] s^2 \\ & + [T_A T_F^2 + 2 \xi T_a (T_F^2 + 2 T_F T_A) + T_a^2 (2 T_F + T_A)] s^3 \\ & + [2 \xi T_a T_A T_F + T_a^2 (T_F^2 + 2 T_F T_A)] s^4 \\ & + T_a^2 T_A T_F^2 s^5 \end{aligned}$$

The steady state gain of the system has been modified from $\frac{\Delta V_R}{1845}$
to $\frac{\Delta V_R (r + 1)}{1845 (1 + N r)}$.

The first order term has been changed from $2 T_F + T_A + 2 \xi T_a$
to $\frac{2 T_F + T_A + 2 \xi T_a + N r T_{\psi}^1}{1 + N r}$

The so-called radome induced time lag, $N r T_{\psi}^1$, can attain values of as
much as 2 to 4 seconds at high altitude conditions. Physically $T_{\psi}^1 = \frac{E}{AE - BC}$

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can be seen from the equations for the airframe to equal ~~error~~. This means that the larger the amount of ϕ required to produce a unit $\dot{\gamma}$, the more the guidance system is affected by radome error slopes. For a given airframe this occurs as the altitude is increased.

The actual effect of r depends on its sign. If it is negative it might cause system instability, if it is positive, the system is slowed up and also causes system instability if the autopilot is not designed properly.

The possible means of coping with r are the following:

- (1) Discredit large r 's by statistics.
- (2) Artificially bias off the negative r 's by feeding into the autopilot a term proportional to $r_{\text{den}} \dot{\psi}$ (explained in section 2.4.2). This results in a situation where only positive r 's need to be dealt with.
- (3) Design a compensating network to stabilize the system for negative r 's. A lag-lead network has been found to be feasible when the flight condition is fixed.
- (4) A combination of (2) and (3).

The advantages and disadvantages of (2) and (3) are:

The advantage of (2) is that it is simple and the resulting system can be extremely stable. The disadvantage is that the system is deliberately slowed down.

The advantage of (3) is that the system can operate with both positive and negative r 's and has $r = 0$ as the nominal condition. The disadvantage is that the parameters of the lag-lead network will probably have to be varied with flight conditions.

The Tartar guidance computer was designed according to (2). The autopilot was designed to accommodate a $(r + r_{\text{max}}) = .08$. This autopilot along with the guidance filter of $\frac{1}{(1 + 2.5s)^2}$ however results in a situation where the system might possibly tolerate negative r 's. This means that $2T_f + T_A + 2.5T_{\text{den}}$ is sufficiently greater than NrT_f . Or in the Bode plot of Figure 9, if the phase is changed by 180° the system is still stable for certain values of r . It might be possible therefore to use something less than $r_{\text{max}} \dot{\psi}$ feedback.

4.2.3 Guidance Filter

The guidance filter has been shown to be

$$G_g(s) = \frac{1}{(1 + 2.5s)(1 + 2.5s)}$$

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In order to show the need for a second order filter, the total transfer function between ϵ_{noise} and m_c will be derived,

$$\begin{aligned}\frac{\epsilon'}{\epsilon_{noise}} &= \frac{1}{1 + \frac{4K}{s^2} + \frac{5K}{s}} \\&= \frac{1}{1 + \frac{4K}{s^2} + \frac{5K}{s}} = \frac{s^2 + sK}{s^2 + sK + 4K} \\&= \frac{s(\frac{1}{4} + \frac{5}{4K})}{\frac{s^2}{4K} + \frac{s}{4} + 1}\end{aligned}$$

therefore

$$\frac{\epsilon'}{\epsilon_{noise}} = \frac{s^2(\frac{1}{4} + \frac{5}{4K})}{\frac{s^2}{4K} + \frac{s}{4} + 1}$$

and for ϵ'

$$\frac{\epsilon'}{\epsilon_{noise}} = \frac{s}{\frac{s^2}{4K} + \frac{s}{4} + 1}$$

$$m_{c,RMS} \approx G_g(s) \frac{[s^2(\frac{1}{4} + \frac{5}{4K}) + s] \epsilon_{noise}}{\frac{s^2}{4K} + \frac{s}{4} + 1} = G_g(s) \epsilon_{noise}$$

In order for $m_{c,RMS}$ to remain finite when ϵ_{noise} is white noise, $G_g(s)$ must be at least second order. The .25 second values were determined experimentally. These values can be subject to change with changes in the estimates of ϵ_{noise} expected.

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4.2.4 Effect of Head Rate Gyro Limitations.

The head rate gyro null value appears directly as a guidance signal bias. The equivalent N_c due to this bias can be computed simply

$$n_c = \frac{A V_R}{1845} \dot{\theta}_B$$

The g sensitivity of the head rate gyro has the effect of introducing another feedback loop.

The guidance signal for $r = 0$ is now

$$\dot{\theta}' + \dot{\epsilon}' = \dot{\theta} + \dot{\epsilon} + g n = \dot{\theta} + K n$$

where K is the g sensitivity of the head rate gy. in $^{\circ}/\text{sec}/g$.

since

$$n = \frac{\frac{A V_R}{1845} (\dot{\theta} + K n)}{(1 + T_s)(1 + 2\zeta T_a s + T_a^2 s^2)(1 + T_\theta s)^2}$$

$$\frac{n}{\dot{\theta}} = \frac{\frac{A V_R}{1845}}{1 - \frac{A V_R K}{1845} + (2T_F + 2\zeta T_a + T_\theta)s + \dots}$$

This shows that the guidance gain and all the coefficients of the guidance transfer functions are modified by

$$\text{If } \frac{A V_R}{1845} \approx 4 \text{ and } K = .03 ^{\circ}/\text{sec}/g \text{ this represents } \approx 12\%$$

variation on the gains and coefficients.

4.2.5 Effect of Head Servo Amplifier Bias.

If a bias exists at the input to the servo amplifier there is no effect on the guidance. However during the head positioning phase there is a $H(s)$ feedback from β to servo amplifier input and the effect of this bias is

$$\beta = \frac{K'}{s^2} [L_b - H(s)\beta]$$

where L_b is the unbalance at the grid,

$$\beta = \frac{L_b}{\frac{s^2}{T_R} + H(s)}$$

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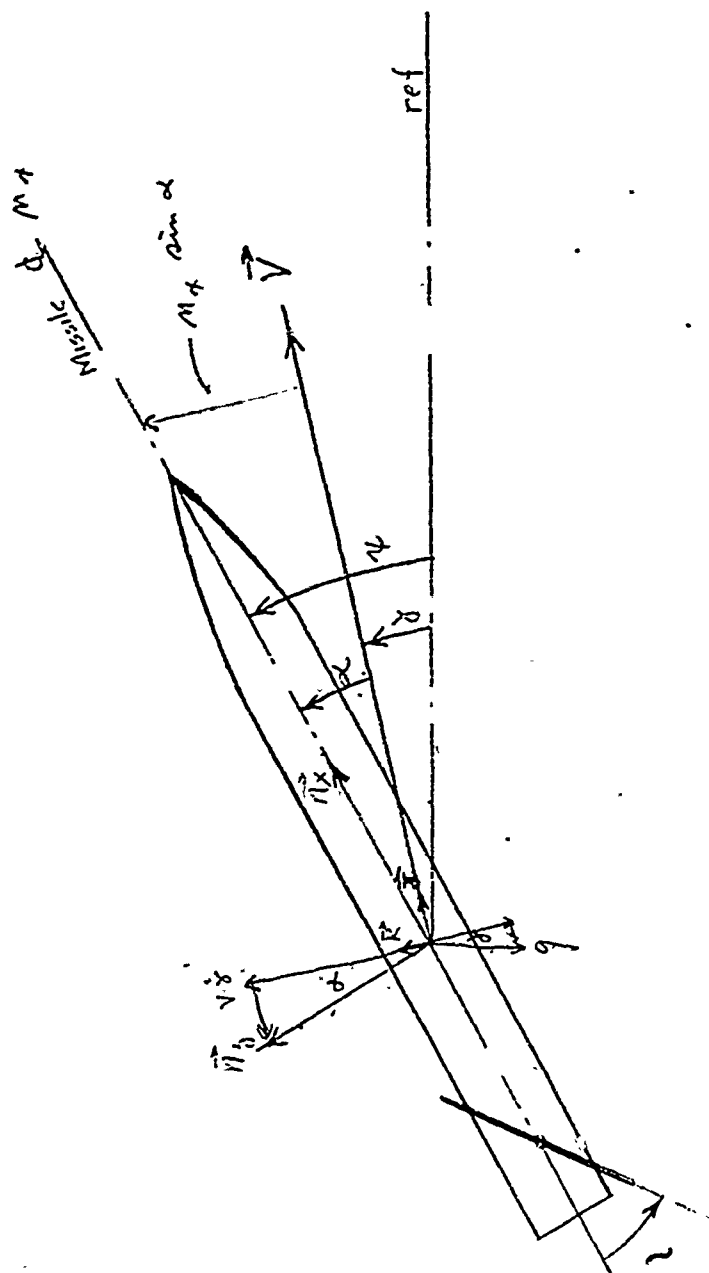
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This shows that the steady state error due to L_b is dependent on the zero frequency gain of $H(s)$.

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Figure 1

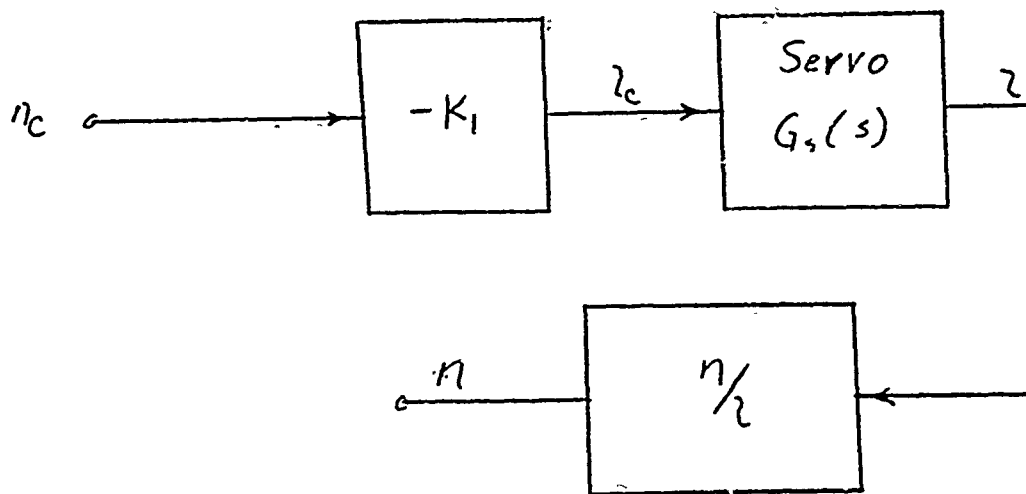


DEFINITION OF ANGLES FOR SINGLE PLANE ANALYSIS

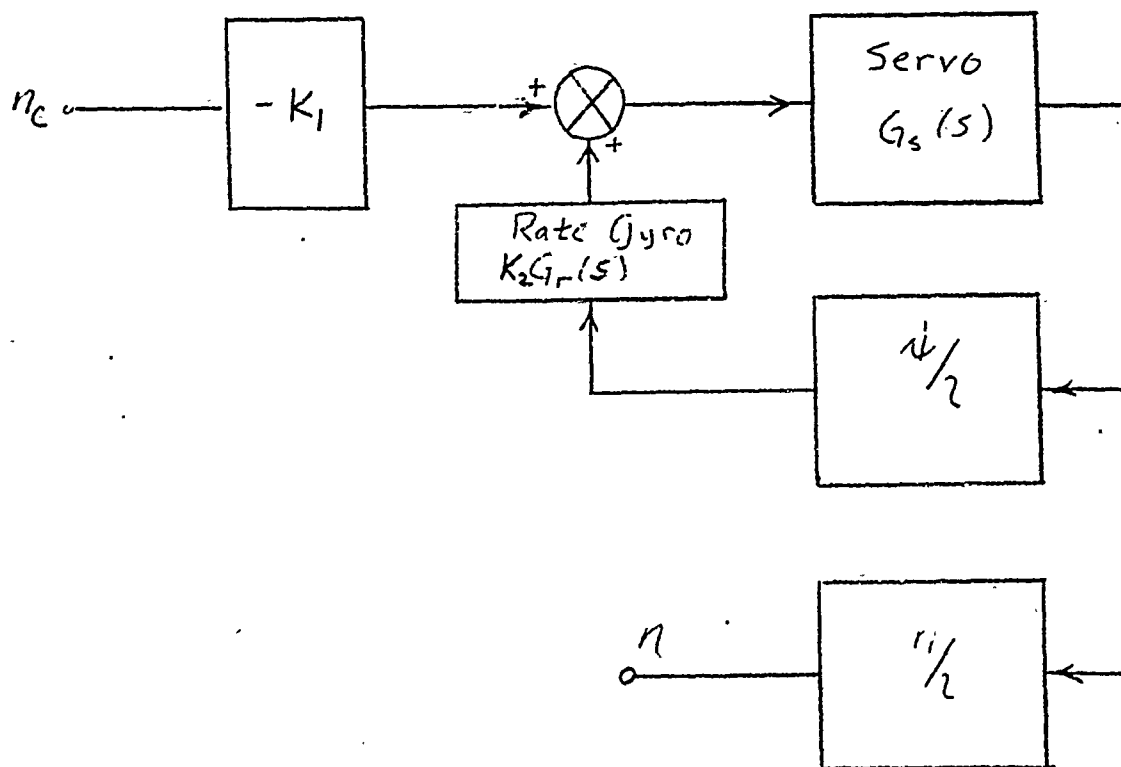
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Figure 2



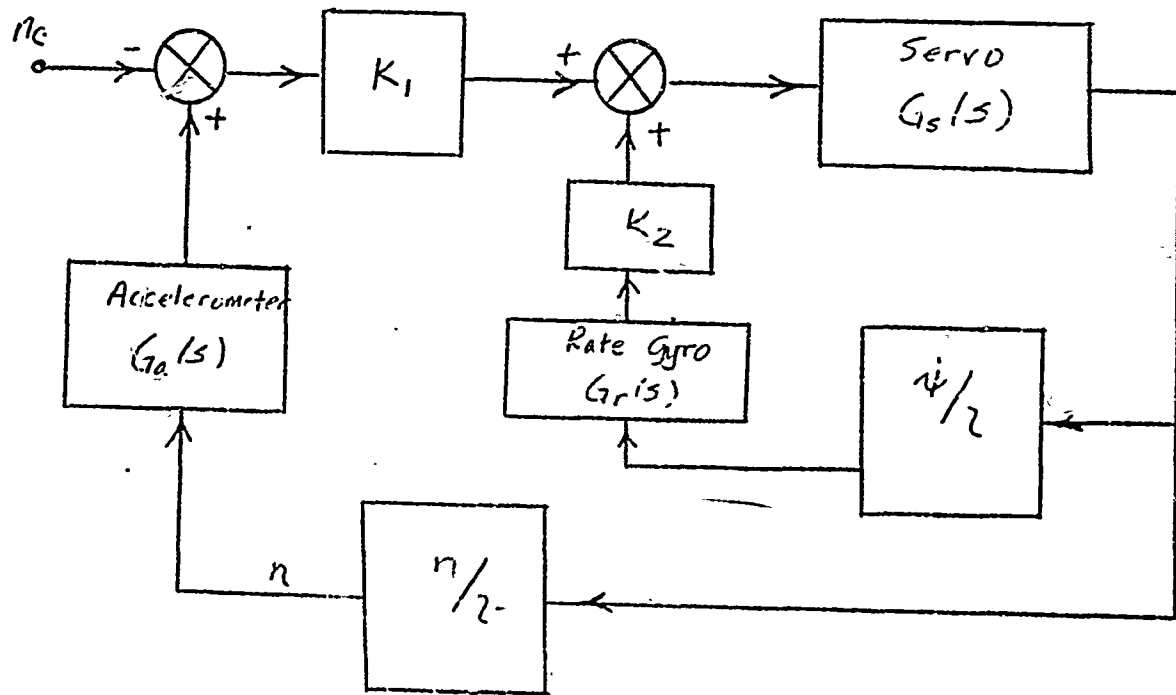
BLOCK DIAGRAM OF TYPE A SYSTEM
Fig. 2 (a)



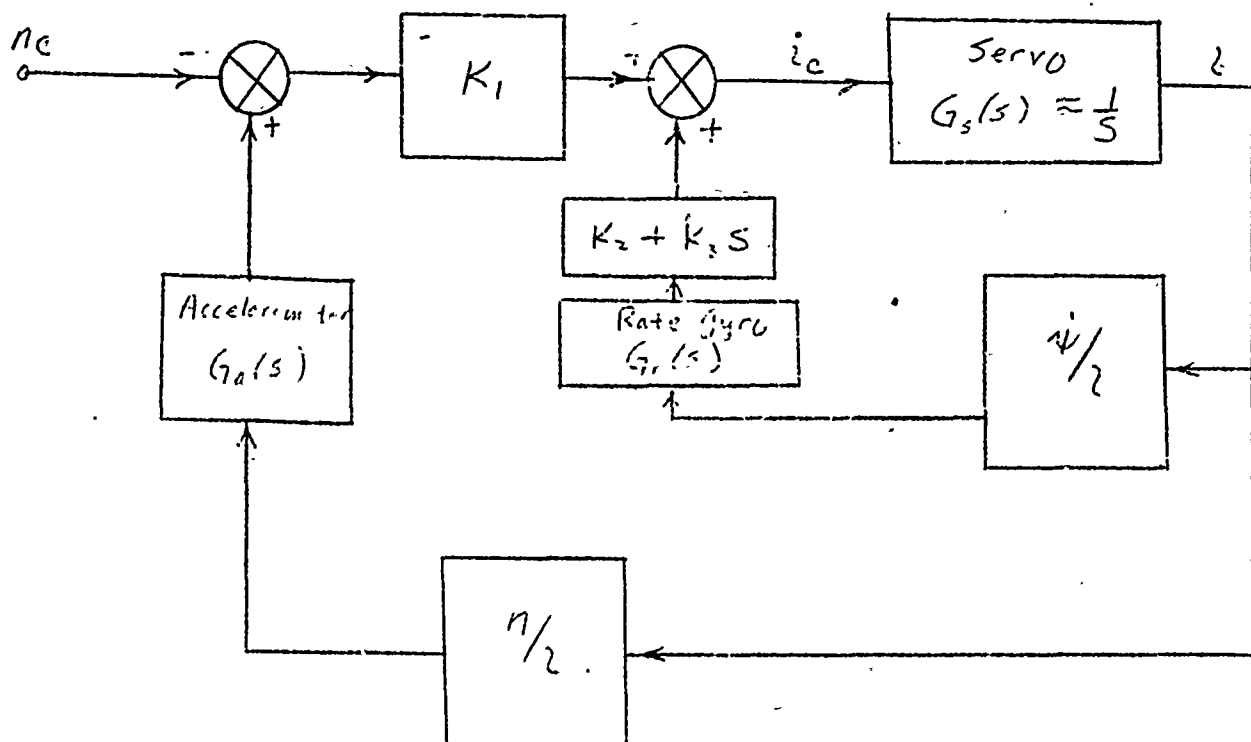
BLOCK DIAGRAM OF TYPE B SYSTEM
Fig. 2 (b)

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BLOCK DIAGRAM OF TYPE C SYSTEM
Fig. 2 (c)

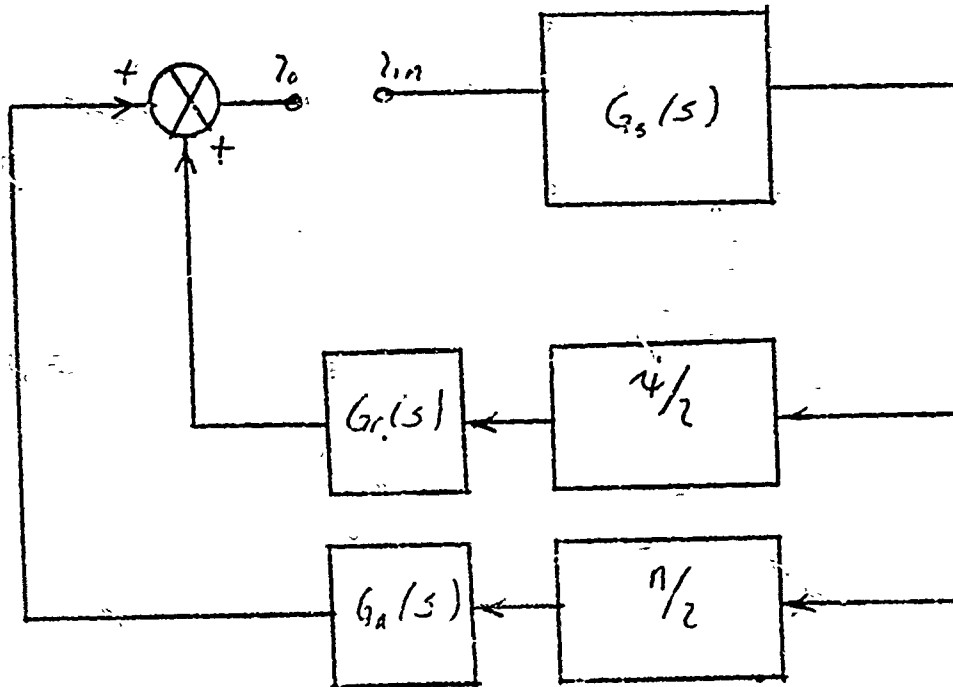


BLOCK DIAGRAM OF TYPE D SYSTEM
Fig. 2 (d)

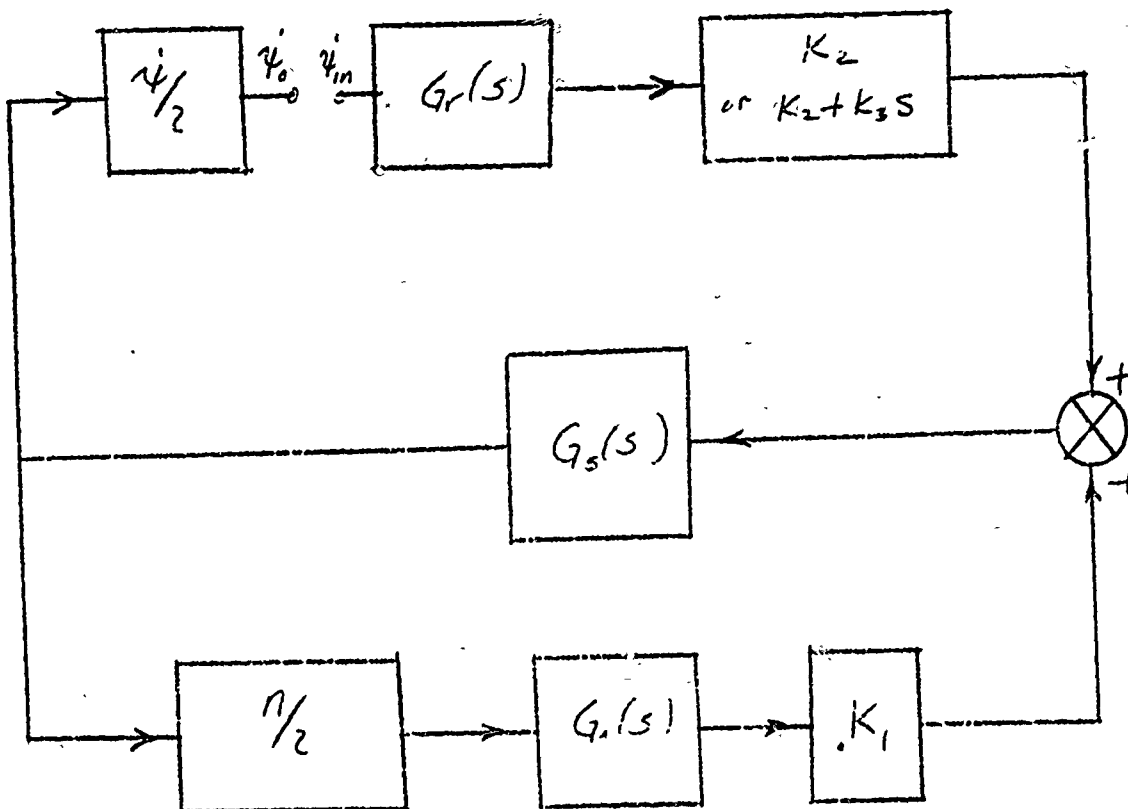
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Figure 3



LOOP OPENED AT WING SERVO
Fig. 3: (a)

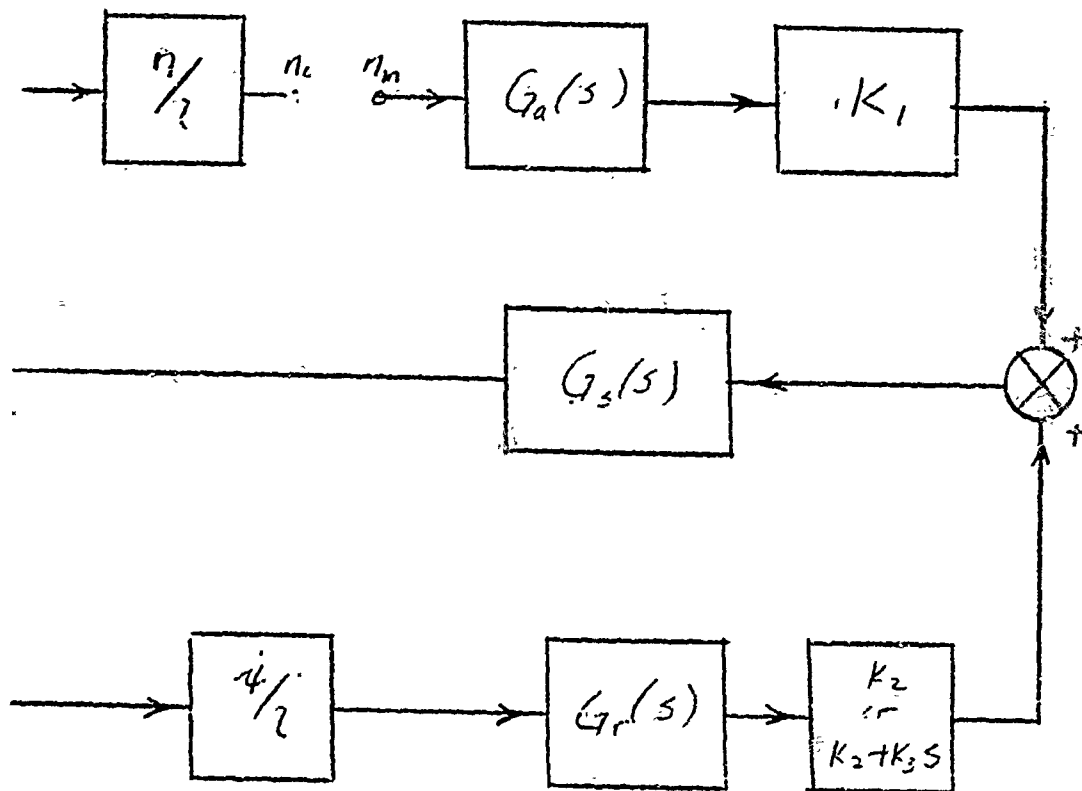


LOOP OPENED AT RATE GYRO
Fig. 3 (b)

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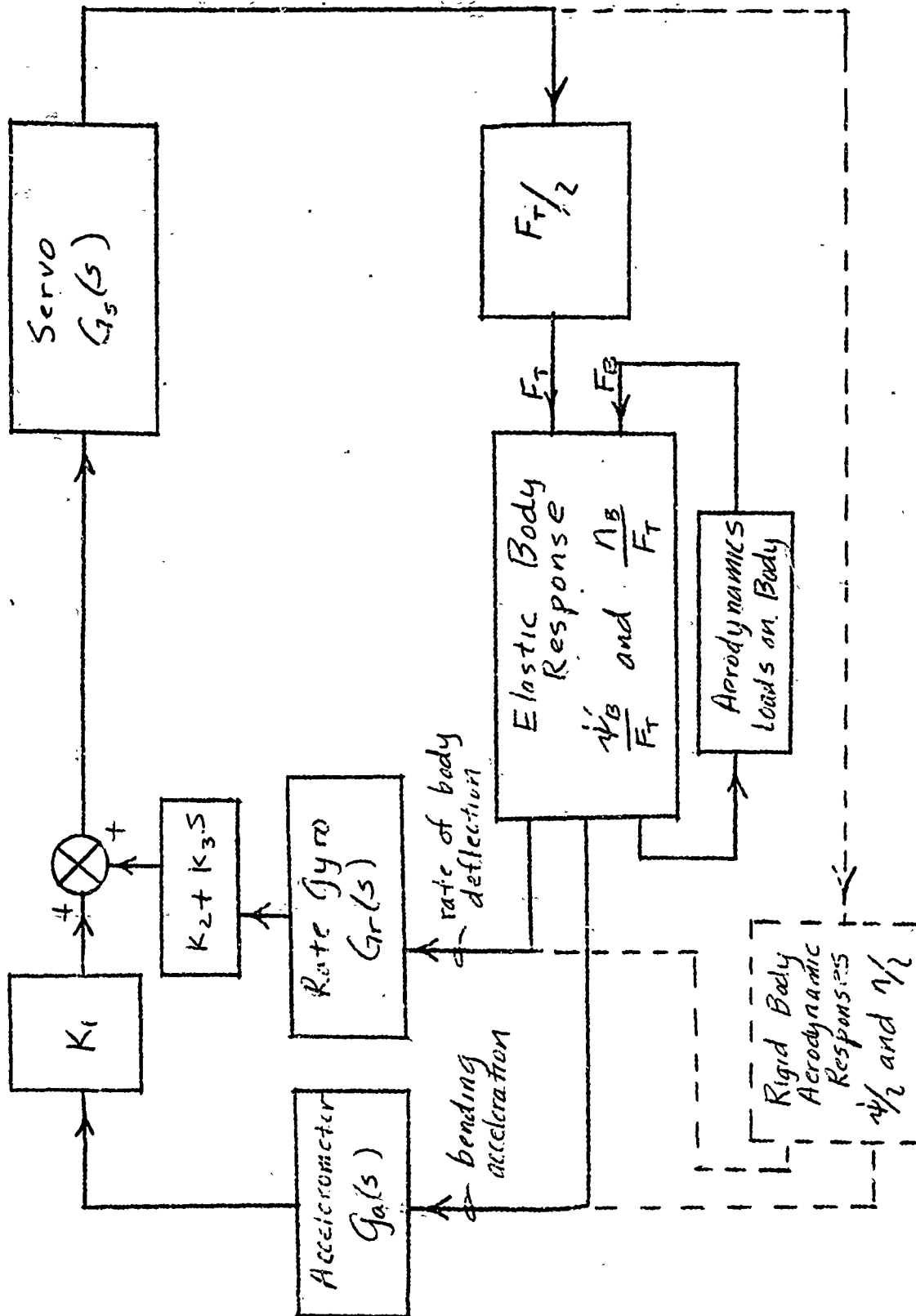


LOOP OPENED AT ACCELEROMETER
Fig. 3 (c)

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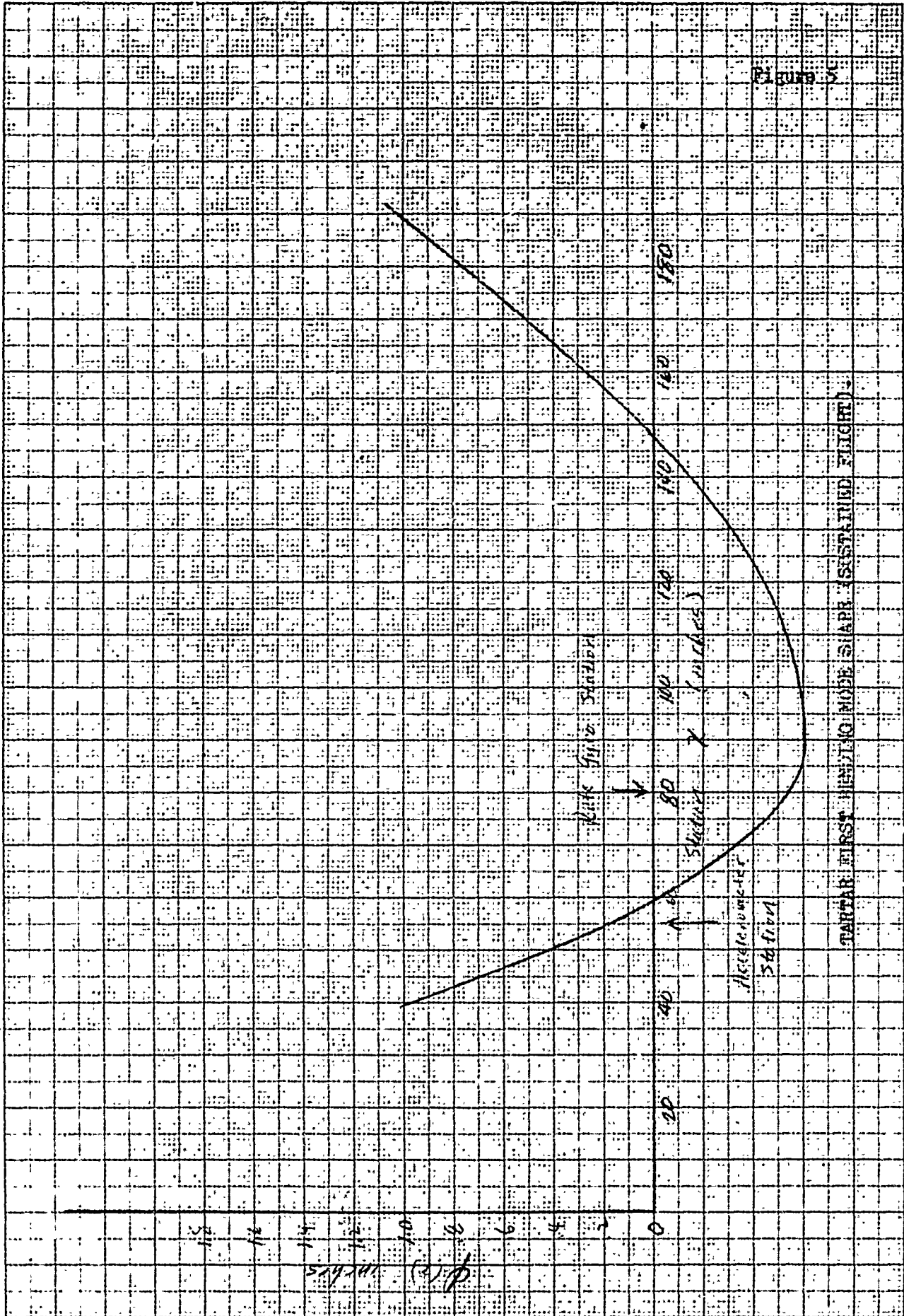
Figure 4



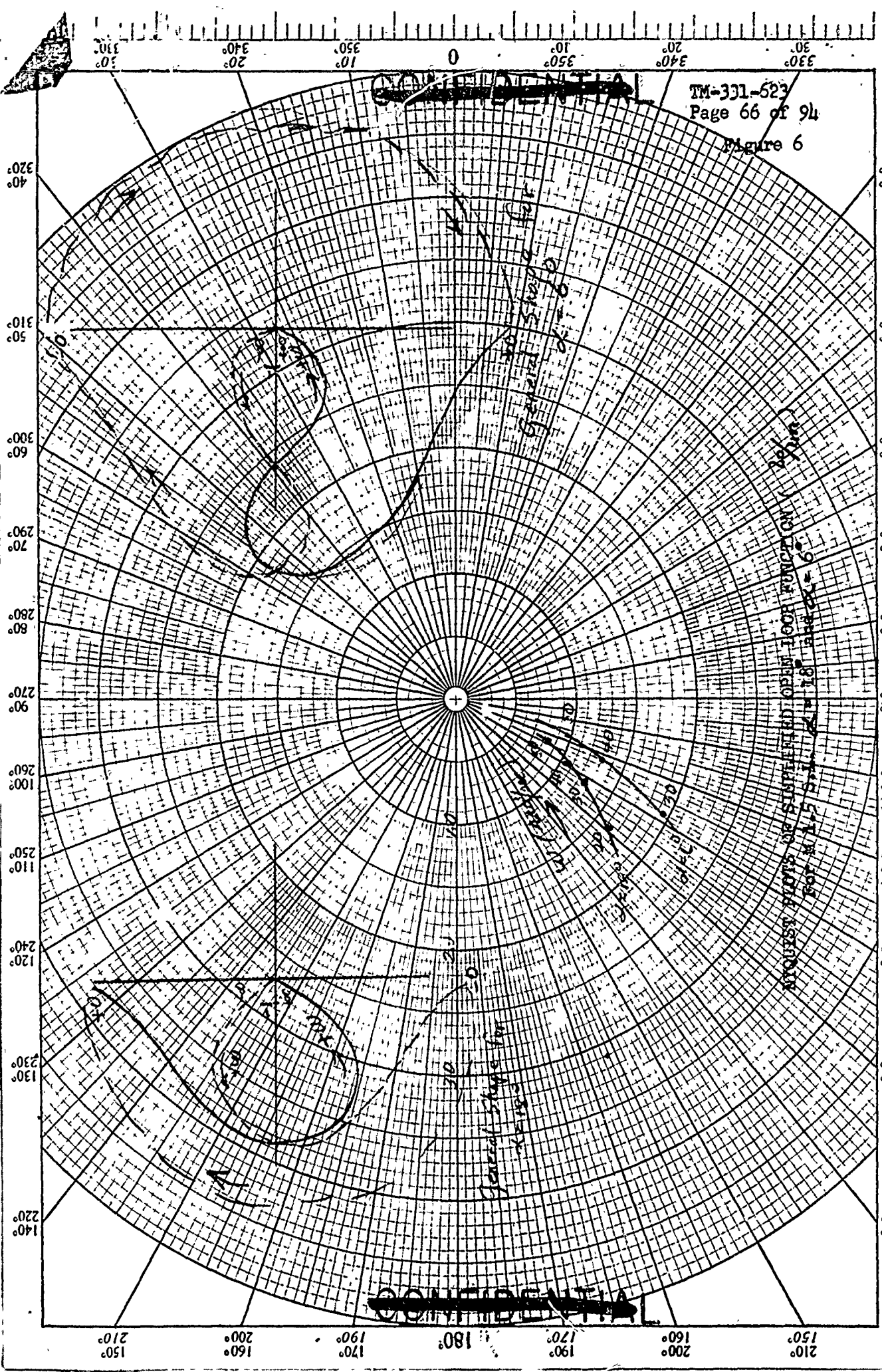
ELASTIC BODY COUPLING LOOP

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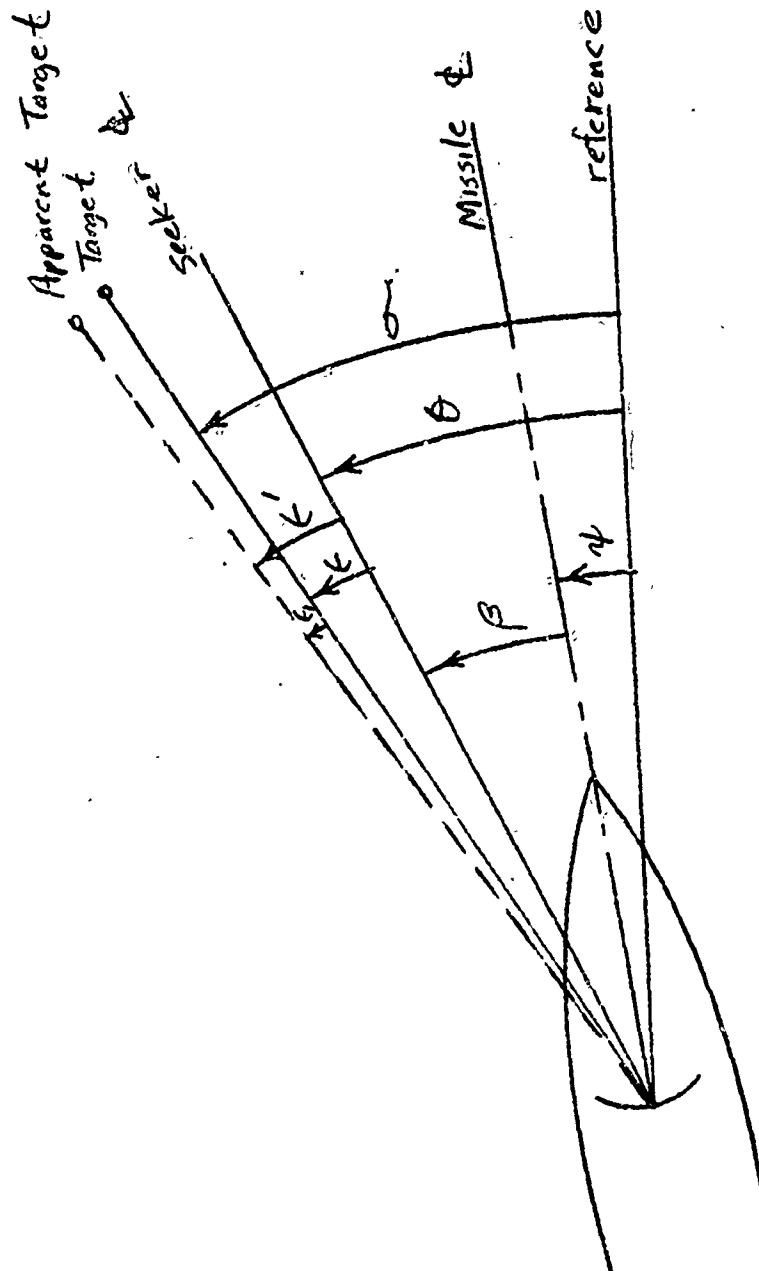
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Figure 6

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KEUFFEL & ESSER CO. MADE IN U.S.A.

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Figure 7



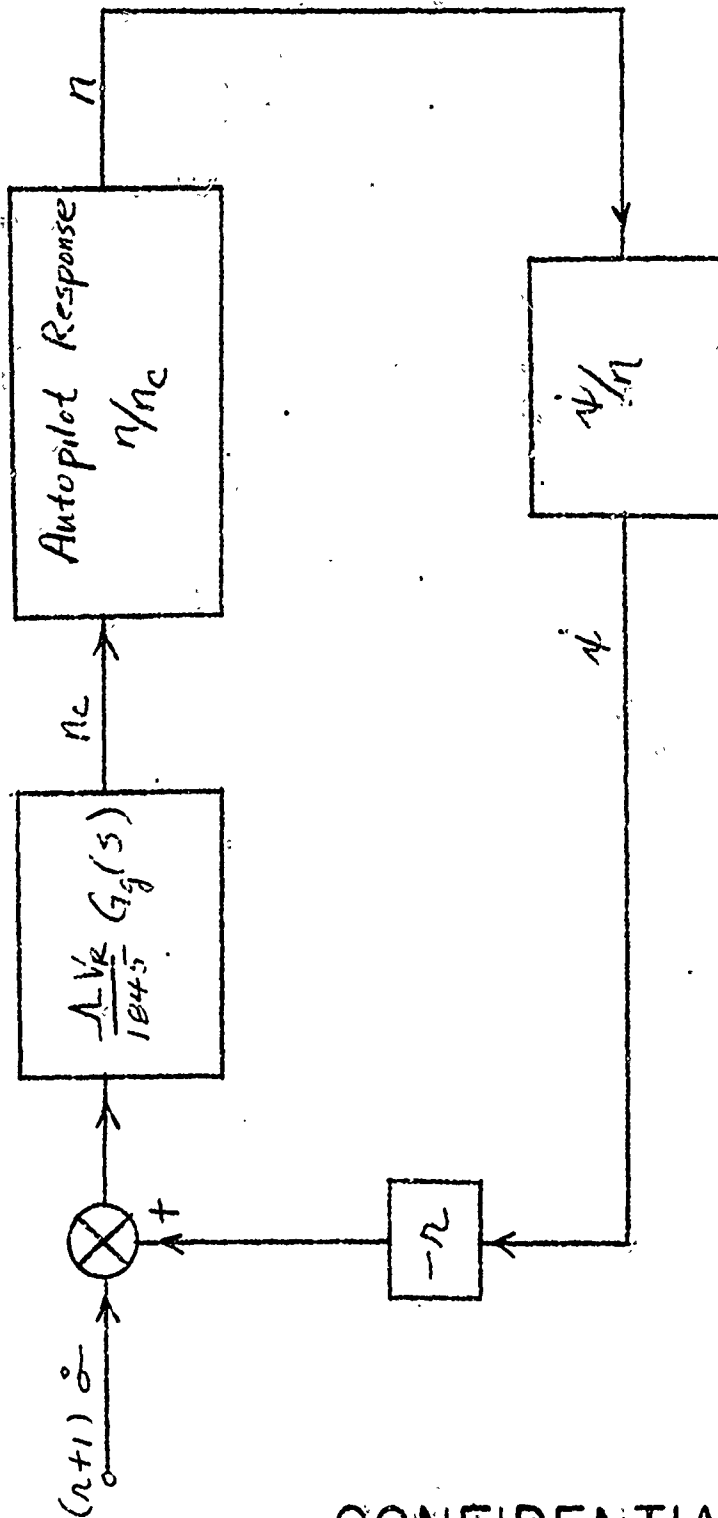
DEFINITION OF ANGLES FOR THE SEEKER

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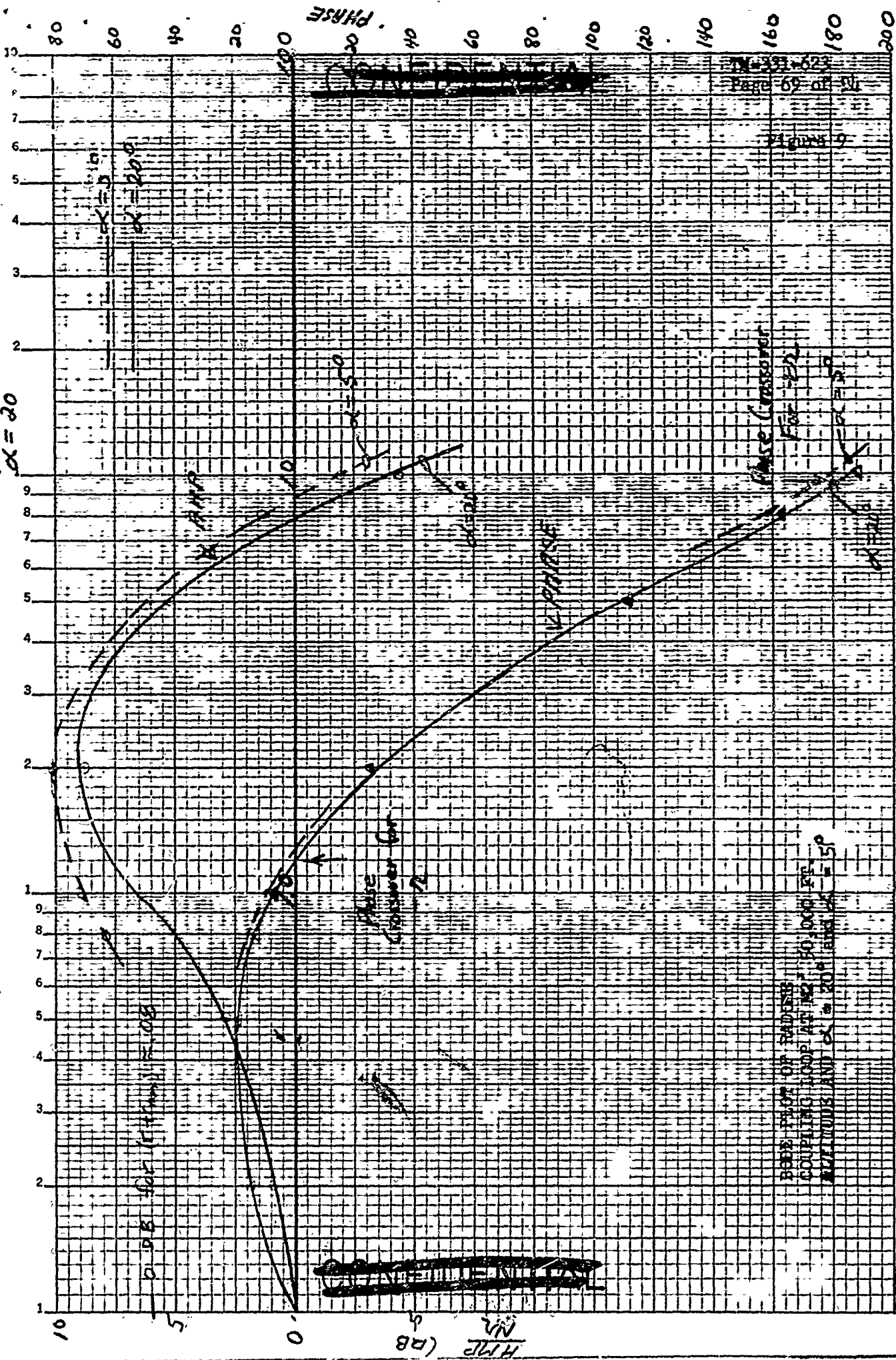
Figure 8



BLOCK DIAGRAM OF RADOME COUPLING LOOP

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M 2 - 50K
 $\phi = 0$
 $\alpha = 20$

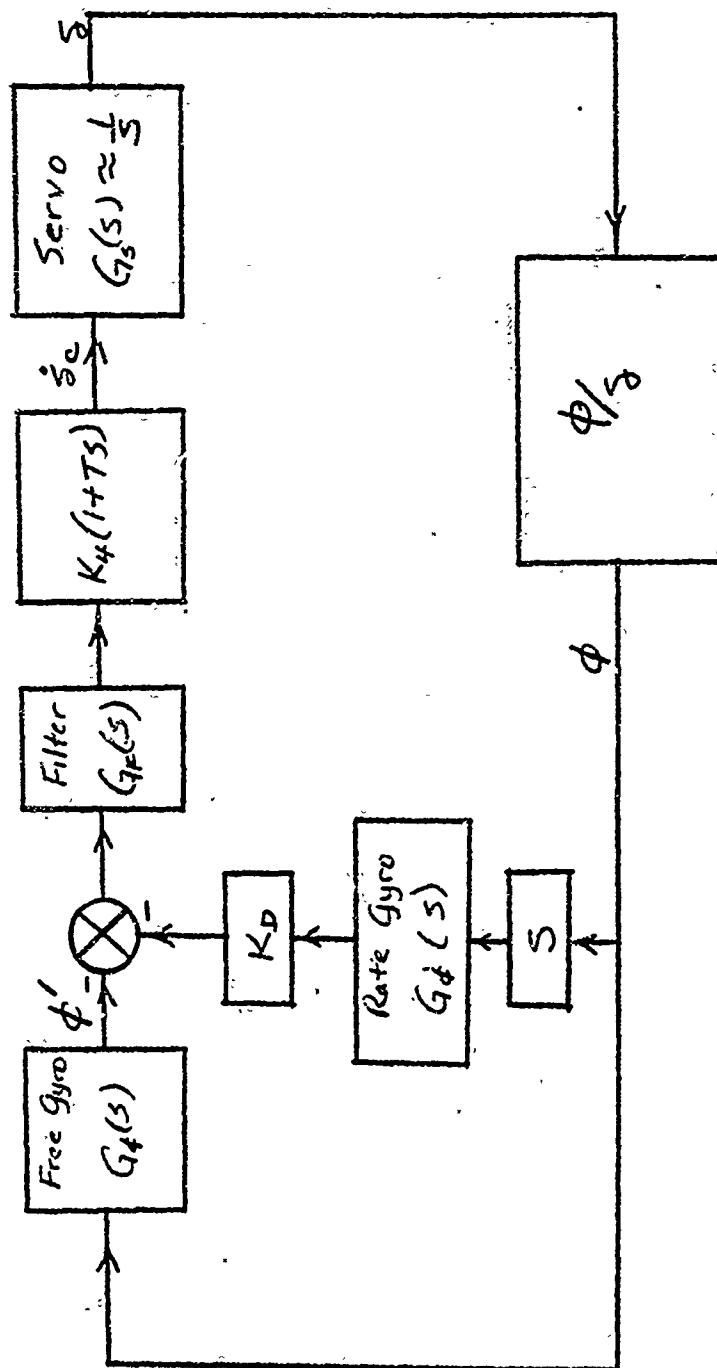


BOB: PLOT OF RADIIUS
COUPLING LOOP AT $\alpha = 20, 100, 180$
FOR $\alpha = 20, 100, 180$

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Figure 10

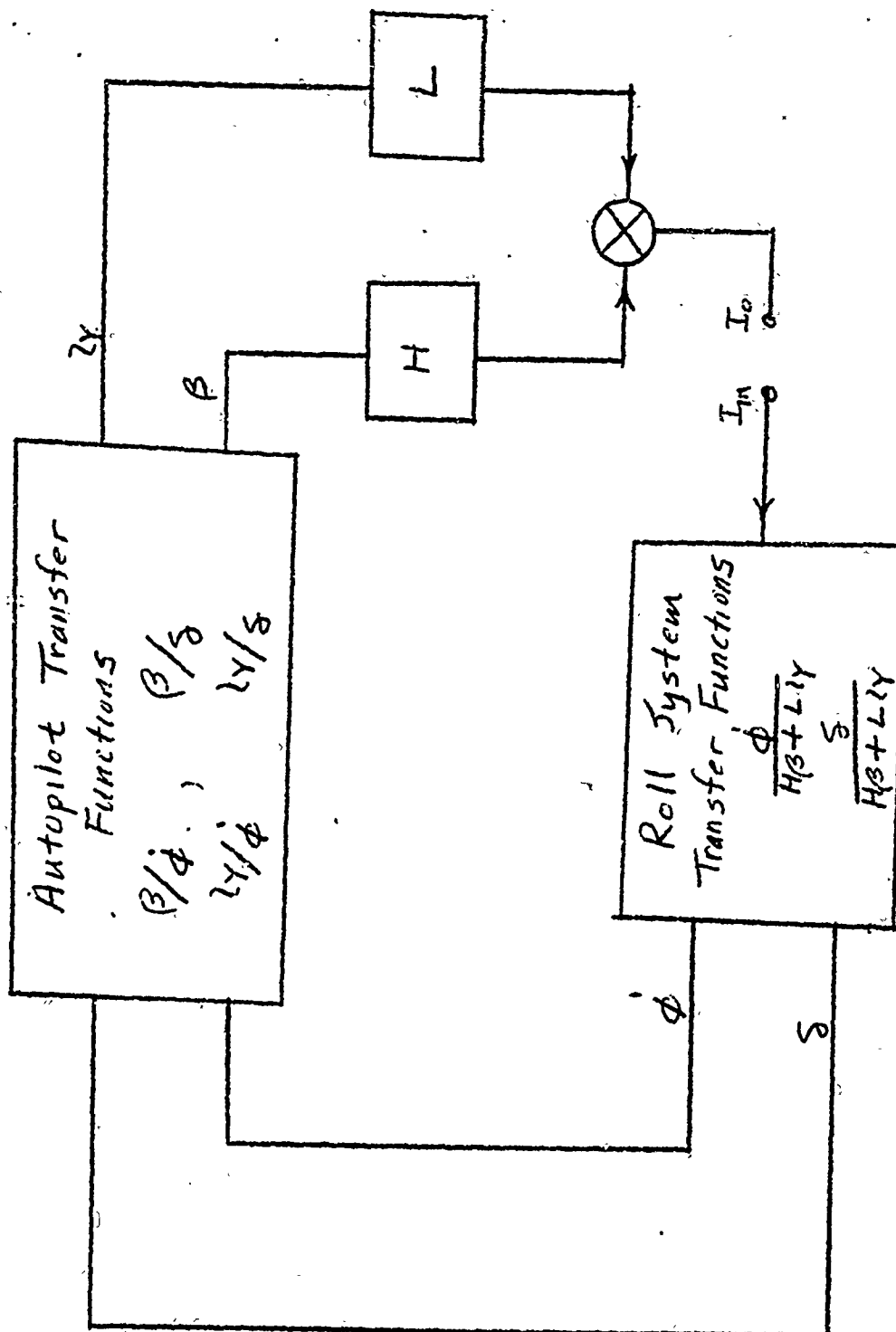


SIMPLIFIED BLOCK DIAGRAM OF ROLL SYSTEM

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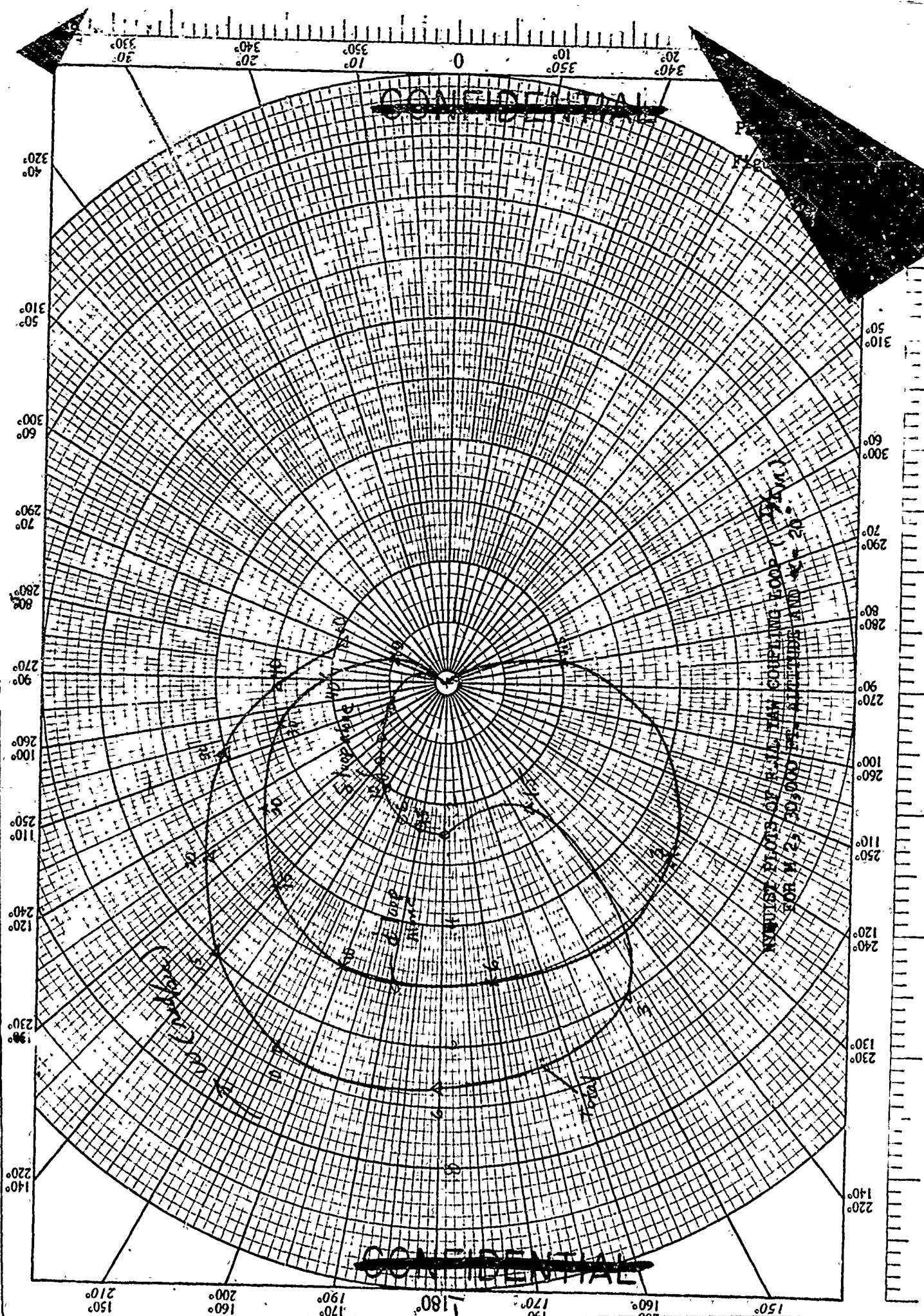
Figure 11



BLOCK DIAGRAM OF COMPLETE ROLL-YAW SYSTEM

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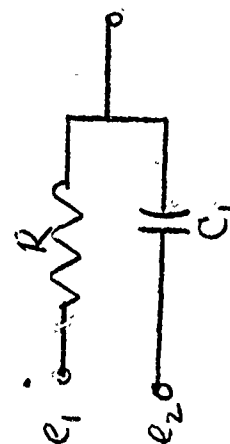
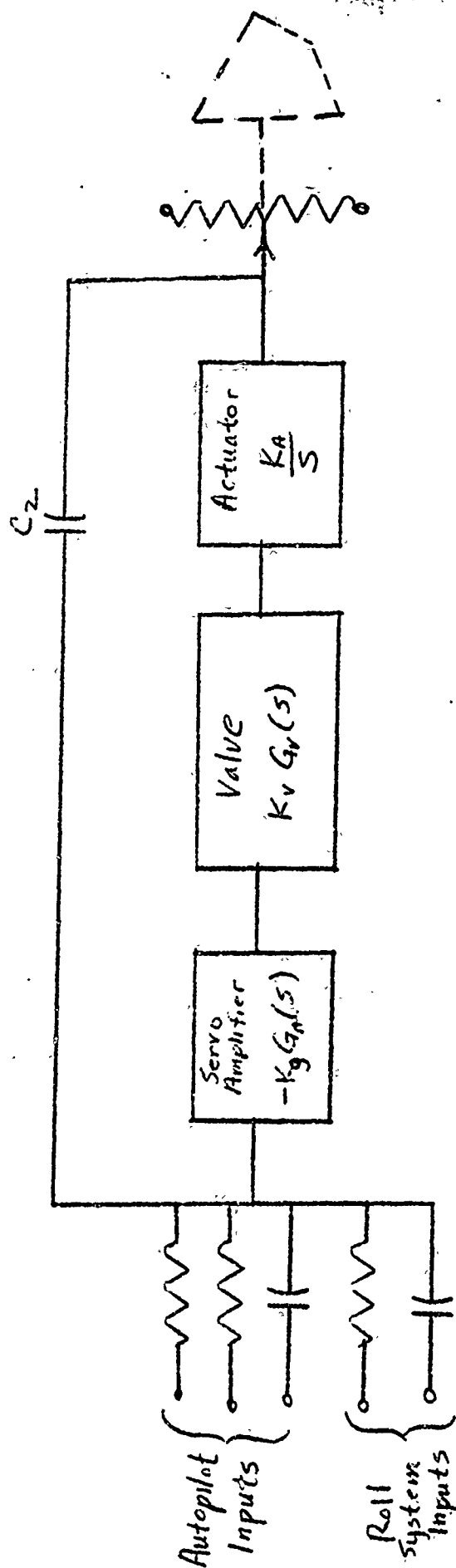
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Figure 13



Equivalent input network

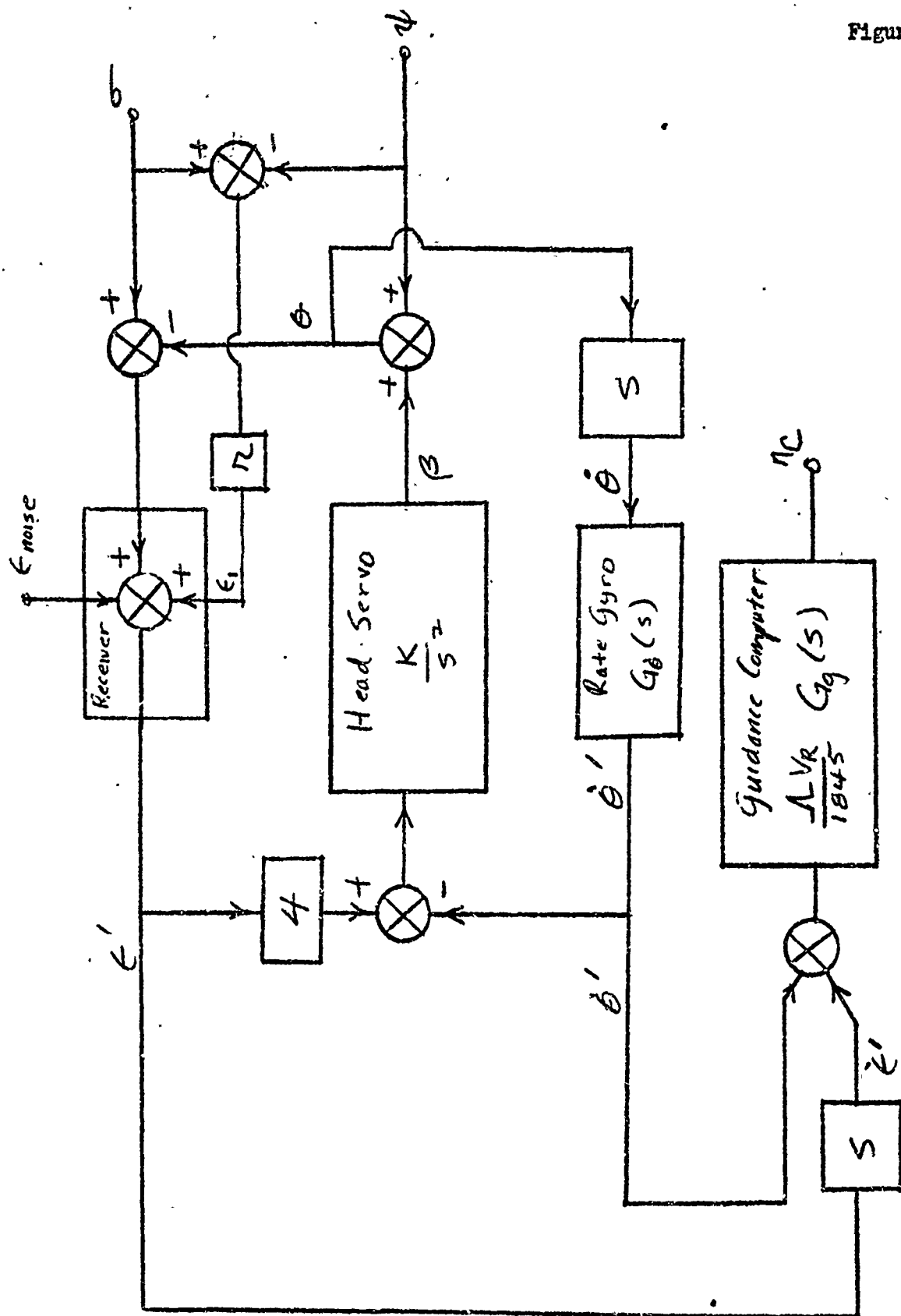
BLOCK DIAGRAM OF CONTROL SURFACE SERVO

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Figure 14



BLOCK DIAGRAM OF GUIDANCE SYSTEM

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APPENDIX I*
DERIVATION OF TRANSFER
FUNCTION FOR ELASTIC BODY

The equation for an elastic beam is

$$(1) \quad \frac{d^2}{dx^2} E(x) I(x) \frac{d}{dx} y(x,t) = -m(x) \frac{d^2 y}{dt^2}(x,t)$$

Assume the solution is the product of a function of x and t

$$y(x,t) = \phi(x) F(t)$$

Substituting into equation (1)

$$F(t) \frac{d^2}{dx^2} E(x) I(x) \frac{d}{dx} \phi(x) = -m(x) \phi(x) \frac{d^2}{dt^2} F(t)$$

dividing through by $F(t) m(x) \phi(x)$

$$\frac{1}{m(x)} \frac{d^2}{dx^2} E(x) I(x) \frac{d}{dx} \phi(x) = - \frac{1}{F(t)} \frac{d^2}{dt^2} F(t)$$

Since $\phi(x)$ does not vary with t and $F(t)$ does not vary with x both sides of the equation must be equal to a constant

$$- \frac{1}{F(t)} \frac{d^2}{dt^2} F(t) = +\omega^2$$

$$\frac{1}{m(x)} \frac{d^2}{dx^2} E(x) I(x) \frac{d}{dx} \phi(x) = +\omega^2$$

* Appendix I Reference: Convair, San Diego, memo 2U-7-048 K. Kachigan - "The General Theory and Analysis of a Flexible Bodied Missile With Autopilot Control."

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The following two equations results

$$(3) \quad \frac{d^2 F}{dt^2} + \omega^2 F = 0$$

$$(4) \quad \frac{d^2}{dx^2} E(x) F(x) \frac{d^2 \phi}{dx^2} - m(x) \omega^2 \phi = 0$$

The solution to equation 3 is of the form

$$F = A \cos \omega t + B \sin \omega t$$

where ω can take any value.

The solutions to equation 4, however, is limited to a set of discrete values of ω .

$$\omega = \omega_1, \omega_2, \omega_3, \dots$$

ω_1 corresponds to the frequency of the first mode, ω_2 to the second, etc.

The functions $\phi_i^{(n)}$ corresponding to the n^{th} mode describes the shape of the n^{th} mode.

The general solution for U can be written as the sum of the individual solutions.

$$U(x, t) = F(t) \phi(x) = \sum_{i=0}^N F_i(t) \phi_i(x)$$

The functions $\phi_i(x)$ are orthogonal in the interval $(0, L)$ with respect to the function $m(x)$

$$\int_0^L m(x) \phi_i(x) \phi_j(x) dx = 0 \quad \text{when } i \neq j$$

With an arbitrary input $W(x, t)$ equation (1) becomes:

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$$(5) \quad \frac{\partial^2}{\partial x^2} E(x) I(x) \frac{\partial}{\partial x^2} U(x, t) = -m(x) \frac{\partial^2 U(x, t)}{\partial t^2} + w(x, t)$$

The general solution of the reduced equation has been found to be

$$U(x, t) = \sum_{i=0}^n F_i(t) \phi_i(x)$$

Since $\phi_i(x)$ is computed with the displacement normalized at some station (a) the computed mode shape $\phi_i(x)$ is related to $\phi_i(a)$ by

$$\phi_i(x) = \frac{\phi_i(x)}{\phi_i(a)}$$

$$\begin{aligned} U(x, t) &= \sum_{i=0}^n F_i(t) \phi_i(a) \frac{\phi_i(x)}{\phi_i(a)} \\ &= \sum_{i=0}^n q_i(t) \phi_i(x) \end{aligned}$$

Substituting into equation (5)

$$(6) \quad \sum_{i=0}^n \left(q_i(t) \frac{\partial^2}{\partial x^2} E(x) I(x) \frac{\partial}{\partial x^2} \phi_i(x) = -m(x) \phi_i(x) \frac{\partial^2 q_i(t)}{\partial t^2} \right) + w(x, t)$$

to separate the variables for each it is desirable to resolve $w(x, t)$ into the various modes

$$w(x, t) = \sum_{i=0}^n R_i m(x) \phi_i(x)$$

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multiplying by $\phi_j(x)$ and integrating from 0 to L

$$\int_0^L \phi_j(x) w(x,t) dx = \int_0^L \phi_j \sum_{i=0}^{\infty} R_i m(x) \phi_i(x) dx$$

only term with $j=i$ will remain

$$\int_0^L \phi_j(x) w(x,t) dx = R_j \int_0^L m(x) [\phi_j(x)]^2 dx$$

$$R_j = \frac{\int_0^L \phi_j(x) w(x,t) dx}{\int_0^L m(x) [\phi_j(x)]^2 dx}$$

The numerator is called the generalized force and the denominator the generalized mass

$$R_j = \frac{Q_j}{m_j}$$

Equation (6) becomes

$$(7) \quad \sum_{i=0}^n \left(q_i(t) \frac{d^2}{dt^2} E(x) I(x) \frac{d}{dx^2} \phi_i(x) \right) = -m(x) \phi_j(x) \frac{d^2}{dt^2} w(x,t) + R_j m(x) \phi_j(x)$$

There are n equations of the form

$$(8) \quad \frac{1}{m(x) \phi_j(x)} \frac{d^2}{dt^2} E(x) I(x) \frac{d}{dx^2} \phi_j(x) = -\frac{1}{\phi_j(x)} \frac{d^2 q_j(t)}{dt^2} + \frac{R_j}{\phi_j(x)}$$

or since both sides of the equation can be set equal to a constant, $-\omega_j^2$

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$$(9) \quad \frac{d^2 q_i(t)}{dt^2} + \omega_i^2 q_i(t) = F_i$$

In general there will be a damping term present

$$(10) \quad \frac{d^2 q_i}{dt^2} + 2 \xi \omega_i \frac{dq_i}{dt} + \omega_i^2 q_i = F_i$$

or in operational form

$$q_i = \frac{F_i}{s^2 + 2 \xi \omega_i s + \omega_i^2}$$

and

$$u(x, t) = \sum_{i=0}^n q_i(t) \phi_i(x)$$

for the case where only the solution is expanded only to $n=1$.

$$u(x, t) = q(t) \phi(x)$$

The displacement and station a is

$$u(a, t) = q(t) \phi(a)$$

The angular displacement of station (a) is

$$\left[\frac{\partial u(x, t)}{\partial x} \right]_{x=a} = q(t) \left[\frac{d \phi(x)}{dx} \right]_{x=a}$$

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If a load L is assumed to exist only at station b

H_i becomes

$$H_i = \frac{\varphi_i(b) L}{m_i}$$

If only the first mode is to be considered

$$H = \frac{\varphi(b) L(t)}{m_i}$$

The accelerometer output at station (a) becomes

$$\frac{d^2}{dt^2} U(a, t) = \ddot{q} \varphi(a)$$

or in operational form

$$\text{accelerometer output} = \frac{s^2 L \varphi(b) \varphi(a)}{(s^2 + 2\{\omega s + \omega^2\}) m}$$

The rate gyro output at station (a) becomes

$$\frac{d}{dt} \left[\frac{\partial U(x, t)}{\partial \dot{x}} \right]_{x=a} = \dot{q}(t) \left[\frac{\partial \varphi(x)}{\partial x} \right]_{x=a}$$

in operational form

$$\text{rate gyro output} = \frac{s L \varphi(b) \left(\frac{\partial \varphi(x)}{\partial x} \right)_{x=a}}{(s^2 + 2\{\omega s + \omega^2\}) m}$$

The units used are generally the following

$L = \text{lbs force}$

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$\phi(x)$ inches

λ inches

$$m = \frac{\text{lb force}}{\text{in./sec}^2} \text{ in}^2 = \text{lb in sec}^2$$

The accelerometer output in g's therefore becomes

$$n(g/a) = \frac{S^2 (1/\text{sec}^2) L (\text{lb}) \phi(b) (\text{in}) \phi(a) (\text{in})}{(S^2 + 2 \{ \omega S + \omega^2 \} (1/\text{sec}^2) 32.2 (ft/\text{sec}^2/g) 12 (\frac{\text{in}}{\text{ft}}) m (\text{lb in sec}^2))}$$

The rate gyro output becomes

$$\dot{\psi} (\text{deg/sec}) = \frac{S (1/\text{sec}) L (\text{lb}) \phi(b) (\text{in}) \left(\frac{d\phi(a)}{dx} \right)_{x=a} (\text{rad}) 57.3 (\text{deg/rad})}{(S^2 + 2 \{ \omega S + \omega^2 \} (1/\text{sec}^2) m (\text{lb in sec}^2))}$$

L for tail deflection loading only becomes

$$L = 1481 \lambda S M^2 C_{N_S} S$$

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APPENDIX II
TO TM-331-623

TM-343-3-1

Description of the Coordinate System to be Used
Hereafter in the Tartar Induced Roll Phenomena
Investigation.

Dated 26 March 1956

By K. Hiroshige

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CONVAIR
A Division of General Dynamics Corporation
(Pomona)

Memo No. TM-343-3-1
Page 1 of 12

Date: 26 March 1956

Subject: Description of the Coordinate System to be Used Hereafter in the Tartar Induced Roll Phenomena Investigation

INTRODUCTION

All dynamics study of the Tartar induced roll phenomena will hereafter use the coordinate system described in this memo. This system differs from that used heretofore in the Tartar studies, from that used by APL in the STV-5 studies, or that used by Convair personnel in the STV-5 studies. It is believed, however, that this is the system that has been agreed to be everyone concerned as the one to use.

Only those features of the system that are pertinent to the dynamics studies to be performed will be presented. Those studies will be restricted to analyzing small perturbations in yaw and roll with fixed pitch conditions. The equations of motion will be written in a body fixed coordinate system. The orientation of this coordinate system in the body will depend on the initial roll attitude for any particular phase of the study. The aerodynamic forces must therefore be resolved to that body fixed coordinate system which will be used for any initial roll attitude. The major portion of this memo will be involved with the stability derivatives required for these aerodynamic forces. The relationship between these stability derivatives and those for the forces resolved along the wind tunnel axis is shown. The equations to be used in the study are also developed.

DESCRIPTION OF COORDINATE SYSTEM

Figure 1 shows the orientation of the coordinate system in the missile body. The origin is at the missile c.g. The sense of positive rotations are also shown. These conform to a right handed system. Also shown are the orientation of the Z axis with respect to the wings for the cases of $\beta = 0$ and $\beta = -45^\circ$.

Figure 2 shows a positive rotation about the Y axis from XYZ to $X'Y'Z'$. If the missile velocity vector is along the X axis as shown, the positive rotation produces a positive angle of attack α . It should be noted that this positive α produces a negative force along the Z axis (F_z). Also with this sign convention, the positive α will produce a negative pitching moment (M_y) for a statically stable configuration.

Figure 3 shows a negative rotation about the Z axis from XYZ to $X'Y'Z'$. If the missile velocity vector is along the X axis as shown, the negative rotation produces a positive side slip angle (β). This positive β produces a negative force along the Y axis (F_y). Positive β will also produce a positive yawing moment (M_z) for a statically stable configuration.

The sign convention for α and β used are consistent with the usual definitions for those quantities

$$(1) \quad \sin \alpha = \frac{w}{V}$$

$$(2) \quad \sin \beta = \frac{v}{V}$$

Prepared by

K. Hiroshige
K. Hiroshige
Sonic Dynamics Group

Figure 4 shows a positive rotation ($\Delta\beta$) about the X axis from XYZ to X'Y'Z' for a missile pitched at an initial angle of attack of α_0 and zero side slip angle. (This would correspond to the conditions under which the wind tunnel tests are run.) After the rotation, α is changed and there is also a resulting β .

The magnitudes of α and β are obtained from the equations (1) and (2) defining α and β . The component of \bar{V} along the Y' axis (u'); and along the Z' axis (w') must therefore be computed.

The transformation matrix for a rotation about the X axis is:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \beta & \sin \beta \\ 0 & -\sin \beta & \cos \beta \end{pmatrix}$$

The components of \bar{V} before the rotation were as follows:

$$u = V \cos \alpha_0$$

$$v = 0$$

$$w = V \sin \alpha_0$$

After the rotation the components are as follows:

$$u' = V \cos \alpha_0$$

$$v' = V \sin \alpha_0 \sin \beta$$

$$w' = V \sin \alpha_0 \cos \beta$$

Using the definitions for α and β (equations 1 and 2)

$$(3) \quad \sin \alpha = \frac{u'}{V} = \sin \alpha_0 \cos \beta$$

$$(4) \quad \sin \beta = \frac{v'}{V} = \sin \alpha_0 \sin \beta$$

In the dynamics studies, the β will be assumed small and β will vary a small amount $\Delta\beta$ from the initial orientation of the Z axis.

The equations (3) and (4) become

$$(5) \quad \alpha = \alpha_0$$

$$(6) \quad \beta = \sin \alpha_0 \Delta\beta$$

It should be noted that α and β are in the planes of Z', \bar{V} and Y', \bar{V} , respectively.

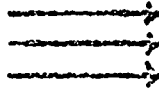
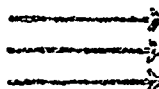
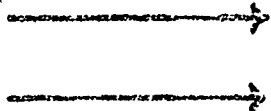
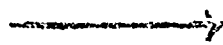
Figure 5 shows the sign conventions for positive deflections of the tail surfaces. In the $\beta = 0$ roll attitude positive δ produces negative F_z and negative M_y . Positive δ produces a positive F_y and a negative M_z .

In the $\beta = -45$ roll attitude the combination of δ and δ produces a positive F_y and a negative M_z .

Figure 6 shows the differential tail deflections (δ) which will produce a positive rolling moment (M_x). This is defined as positive δ .

SUMMARY OF COORDINATE SYSTEM DESCRIPTION

The information presented thus far can be summarized in the following table.

	$\beta = 0$	$\beta = -45$
δ	Produced by δ rotation about Y axis Produces $-F_z$ Produces $-M_y$ for stable configuration	
β	Produced by δ rotation about Z axis Produces $-F_y$ Produces δM_z for stable configuration	
$\delta \beta$	Produced by δ rotation about X axis. Z axis initially in the plane of undeflected δ wings With an initial α , $\delta \beta$ will produce $\beta \approx \sin \alpha \delta \beta$ α will remain $\approx \alpha$	Produced by δ rotation about X axis. Z axis initially at -45° from that for the $\beta = 0$ case. 
δ (deflections defined looking forward)	Produced by deflecting trailing edge down Produces $-F_z$ Produces $-M_y$ Produces no F_y Produces no M_z	Produced by deflecting trailing edge down and right Produces $-F_z$ Produces $-M_y$ Produces $-F_y$ Produces $-M_z$
δ (deflections defined looking forward)	Produced by deflecting trailing edge left Produces no F_z Produces no M_y Produces δF_y Produces $-M_z$	Produced by deflecting trailing edge down and left Produces $-F_z$ Produces $-M_y$ Produces δF_y Produces $-M_z$
δ	Produced by differential deflections that will produce a positive rolling moment	

STABILITY DERIVATIVES

The stability derivatives required for this study are those for the yaw force, yaw moment, and roll moment. These forces and moments must be referred to that body fixed coordinate system which is used for any particular initial roll attitude considered.

Figure 1 shows the body fixed coordinate system to be used for the case of $\beta_0 = 0$ and $\delta_0 = -45^\circ$.

The following table lists the stability derivatives and the signs expected. The pitch derivatives are included though they are not needed for the studies proposed. The signs for the derivatives can be deduced from the information presented in the coordinate system description. The roll yaw coupling derivatives with respect to tail surfaces were assumed to be primarily from the blanking of the upper surfaces when the missile is at a positive α_0 .

Table of Signs Expected for Stability Derivatives

(Assuming α_0 is positive)

	Stability Derivative	$\beta = 0$	$\beta = -45$
Pitch Coefficients	$C_{N\alpha}$	-	-
	$C_{m\alpha}$	- for stable	- for stable
	$C_{N\beta}$	-	-
	$C_{m\beta}$	0	0
	$C_{N\dot{\alpha}}$	-	-
	$C_{m\dot{\alpha}}$	0	0
Yaw Coefficients	$C_{Y\beta}$	-	-
	$C_{n\beta}$	+ for stable	+ for stable
	$C_{Y\dot{\beta}}$	0	0
	$C_{n\dot{\beta}}$	0	0
	$C_{Y\dot{\alpha}}$	-	-
	$C_{n\dot{\alpha}}$	0	0
Roll Coefficients	$C_{l\beta}$	+	+
	$C_{l\dot{\beta}}$	-	-
Roll Yaw Coupling Coefficients	$C_{l\dot{\alpha}}$	- for stable	- for stable
	$C_{n\dot{\alpha}}$	+	+
	$C_{l\dot{\beta}}$	-	-
	$C_{n\dot{\beta}}$	+	+
	$C_{l\dot{\gamma}}$	+	+

EQUATIONS OF MOTION

The required equations of motion for this study are:

$$(7) F_Y = m (\ddot{v} - p\dot{w} + r\dot{u})$$

$$(8) M_Z = \dot{r} I_Z + (I_Y - I_X) p q$$

$$(9) M_X = \dot{p} I_X$$

Making the following substitutions

$$p = \dot{\beta}$$

$$r = \dot{\psi}$$

$$q = \dot{\theta}_0 \text{ (constant pitch rate associated with constant angle of attack)}$$

$$\dot{v} = V \cos \beta \dot{\beta}$$

$$\text{since } \sin \beta = v/V \text{ (equation 1)}$$

$$w = V \sin \alpha_0 \text{ (equation 2)}$$

$$u = V \cos \alpha_0$$

Equations (7 - 9) become

$$(10) F_Y = m (V \cos \beta \dot{\beta} - \dot{\beta} V \sin \alpha_0 + \dot{\psi} V \cos \alpha_0)$$

$$(11) M_Z = I_Z \dot{\psi} + (I_Y - I_X) \dot{\beta} \dot{\theta}_0$$

$$(12) M_X = I_X \dot{\beta}$$

where F_Y is in lbs; M_Z and M_X in ft. lbs; $\dot{\beta}$, $\dot{\theta}_0$, $\dot{\psi}$ are in rad/sec;
 $\dot{\psi}$ in rad/sec²

The equations for the aerodynamic forces and moments are:

$$(13) F_Y = 1481 \lambda S M^2 (C_{Y\beta} \beta + C_{Y\dot{\beta}} \dot{\beta} + C_{Y\delta} \delta)$$

$$(14) M_Z = 1481 \lambda s d M^2 (C_{M\beta} \beta + C_{M\dot{\beta}} \dot{\beta} + C_{M\delta} \delta)$$

$$(15) M_X = 1481 \lambda s d M^2 (C_{L\delta} \delta + C_{L\dot{\beta}} \frac{\dot{\beta}}{2V} + C_{L\beta} \beta + C_{L\dot{\psi}} \dot{\psi})$$

The i_Y coefficients are defined in the following table.

	$\beta = 0$	$\beta = 45$
i_Y	i_Y'	$i_Y' = 1$
$C_{Y\dot{\beta}}$	$C_{Y\dot{\beta}}'$	$C_{Y\dot{\beta}}' = C_{Y\dot{\beta}}$
$C_{M\dot{\beta}}$	$C_{M\dot{\beta}}'$	$C_{M\dot{\beta}}' = C_{M\dot{\beta}}$
$C_{L\dot{\psi}}$	$C_{L\dot{\psi}}'$	$C_{L\dot{\psi}}' = C_{L\dot{\psi}}$

It should be noted $C(\delta)$ for the $\delta = -45$ case equals .707 $C(\delta)$ for the $\delta = 0$ case.

Redefining the aerodynamic coefficients in the following manner

$$A = 1481 \lambda S M^2 \frac{57.3}{\pi V} C_{Y\delta}$$

$$B = 1481 \lambda S M^2 \frac{57.3}{\pi V} C_{YLY}$$

$$C = 1481 \lambda S M^2 \frac{57.3}{I_Z} C_{N\delta}$$

$$E = 1481 \lambda S M^2 \frac{57.3}{I_Z} C_{MY}$$

$$F = 1481 \lambda S M^2 \frac{57.3}{I_X} \frac{b}{2V} C_{L\delta}$$

$$G = 1481 \lambda S M^2 \frac{57.3}{I_X} C_{L\delta}$$

$$H = 1481 \lambda S M^2 \frac{57.3}{I_X} C_{L\delta}$$

$$L = 1481 \lambda S M^2 \frac{57.3}{I_X} C_{LY}$$

$$M = 1481 \lambda S M^2 \frac{57.3}{\pi V} C_{Y\delta}$$

$$N = 1481 \lambda S M^2 \frac{57.3}{I_Z} C_{N\delta}$$

Equations (10 - 12) become

$$(16) \frac{57.3}{\pi V} F_Y = (A \delta + B_{LY} + M \delta) = \cos \delta \dot{\delta} - \dot{\delta} \sin \alpha_0 + \dot{\psi} \cos \alpha_0$$

$$(17) \frac{57.3}{I_Z} N_Z = C \delta + E_{LY} + N \delta = \ddot{\psi} + \frac{(I_Y I_X)}{I_Z 57.3} \dot{\delta} \dot{\psi}$$

$$(18) \frac{57.3}{I_X} M_Y = G \delta + F \dot{\delta} + H \delta + L_{LY} = \ddot{\delta}$$

For these equations all angles are in degrees.

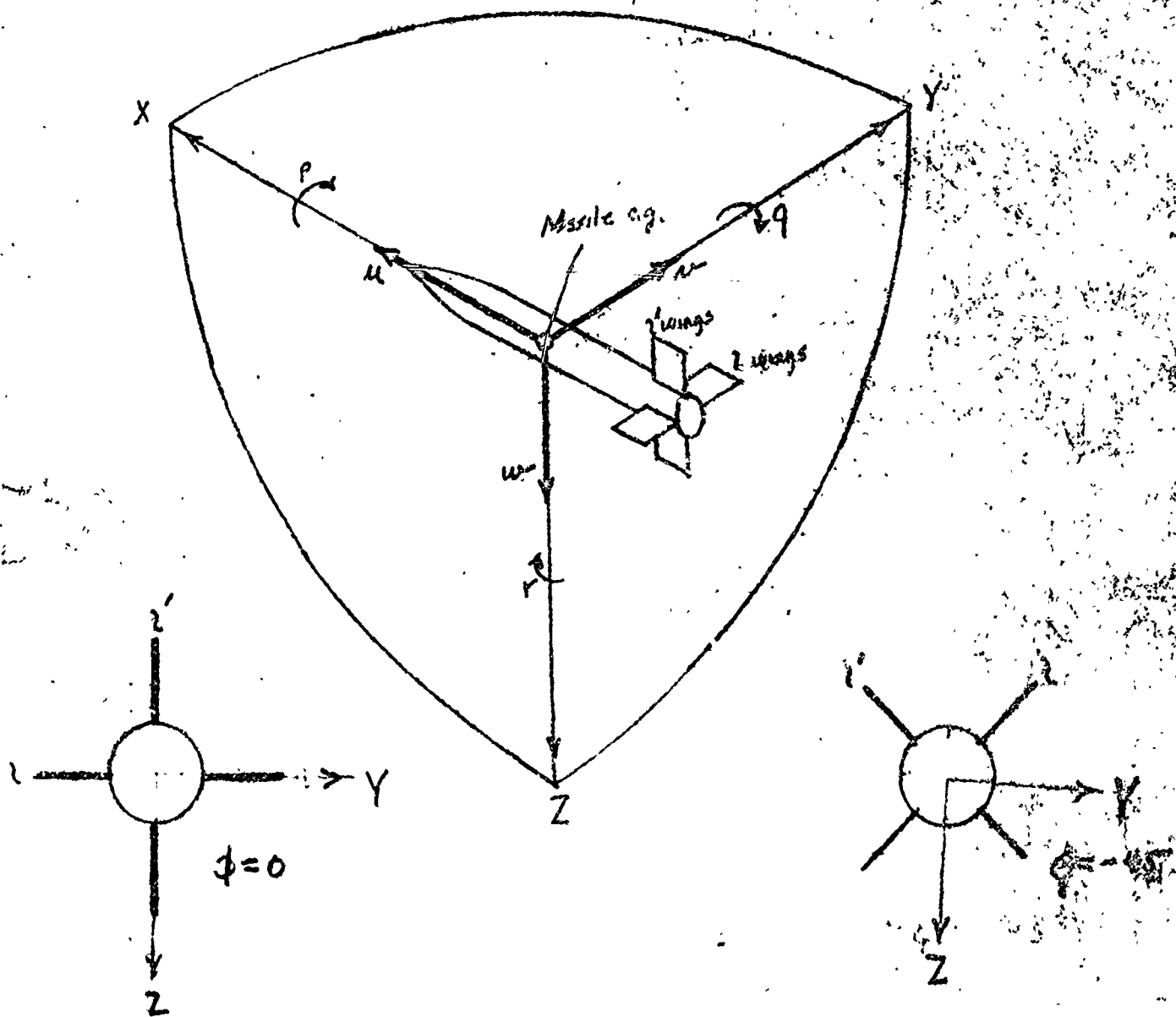
The control equations are

$$\dot{Y}_0 = K_1 (N - N_0) + K_2 \dot{\psi} + K_3 \ddot{\psi}$$

$$\dot{\delta} = -(K_4 + K_5 S + K_6 S^2) \delta$$

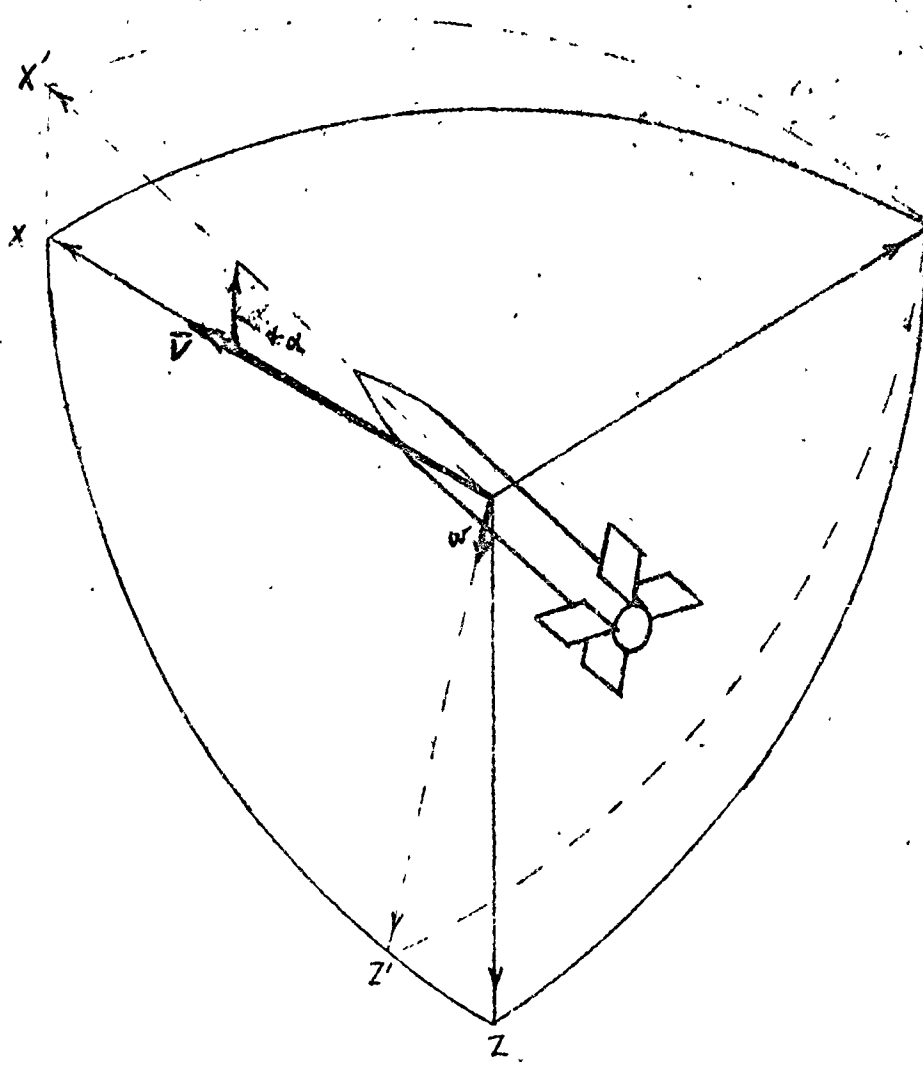
where

$$N (g^2) = \frac{V}{1845} (A \delta + B_{LY} + M \delta)$$



Calculation of Gravity Vector in Figure 1

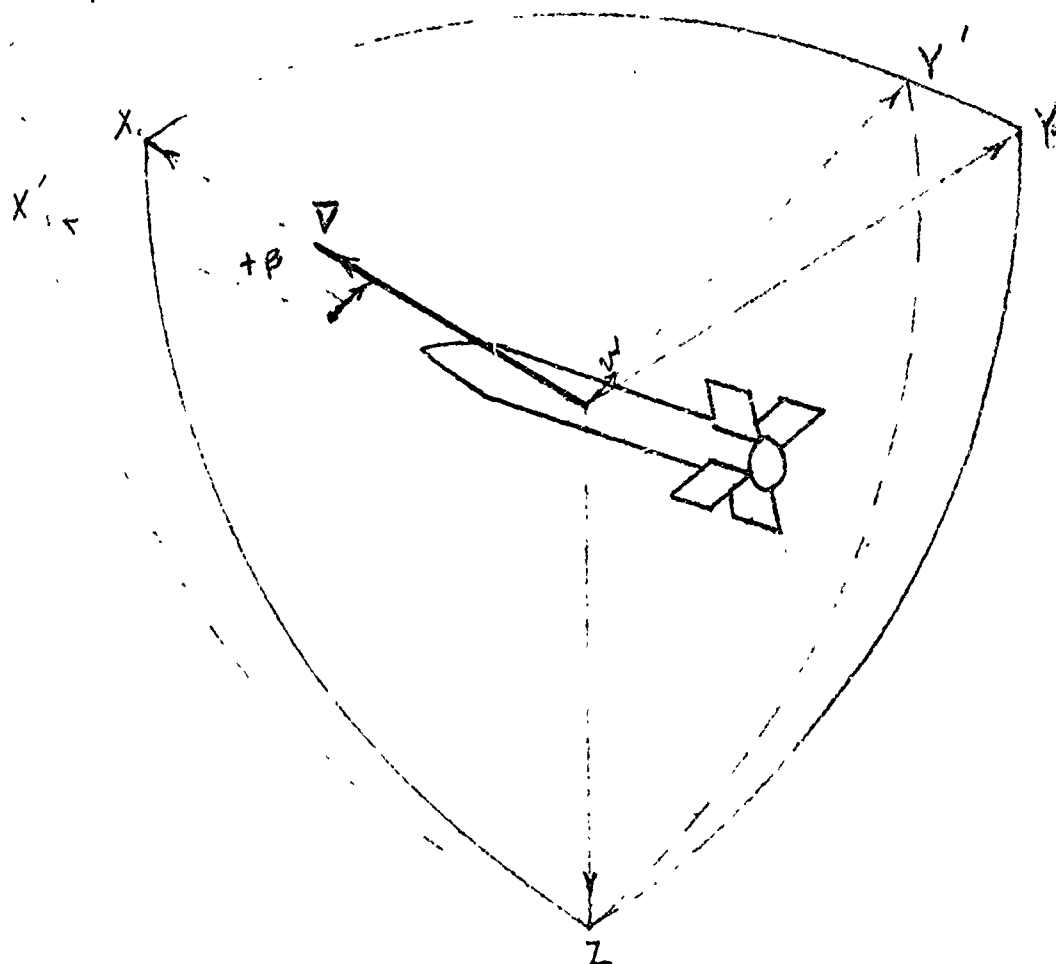
Figure 1



Positive Rotation About Y Axis and the Resulting Angle of Attack

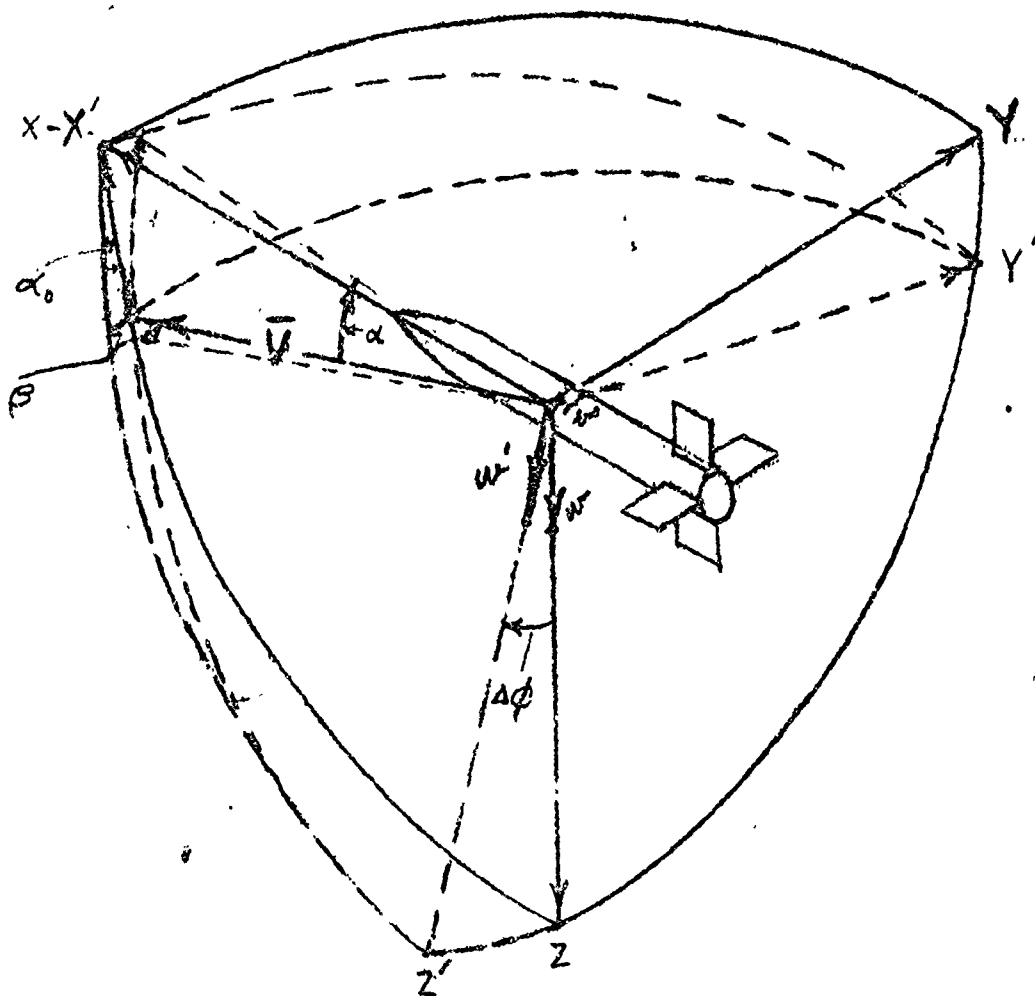
Figure 2





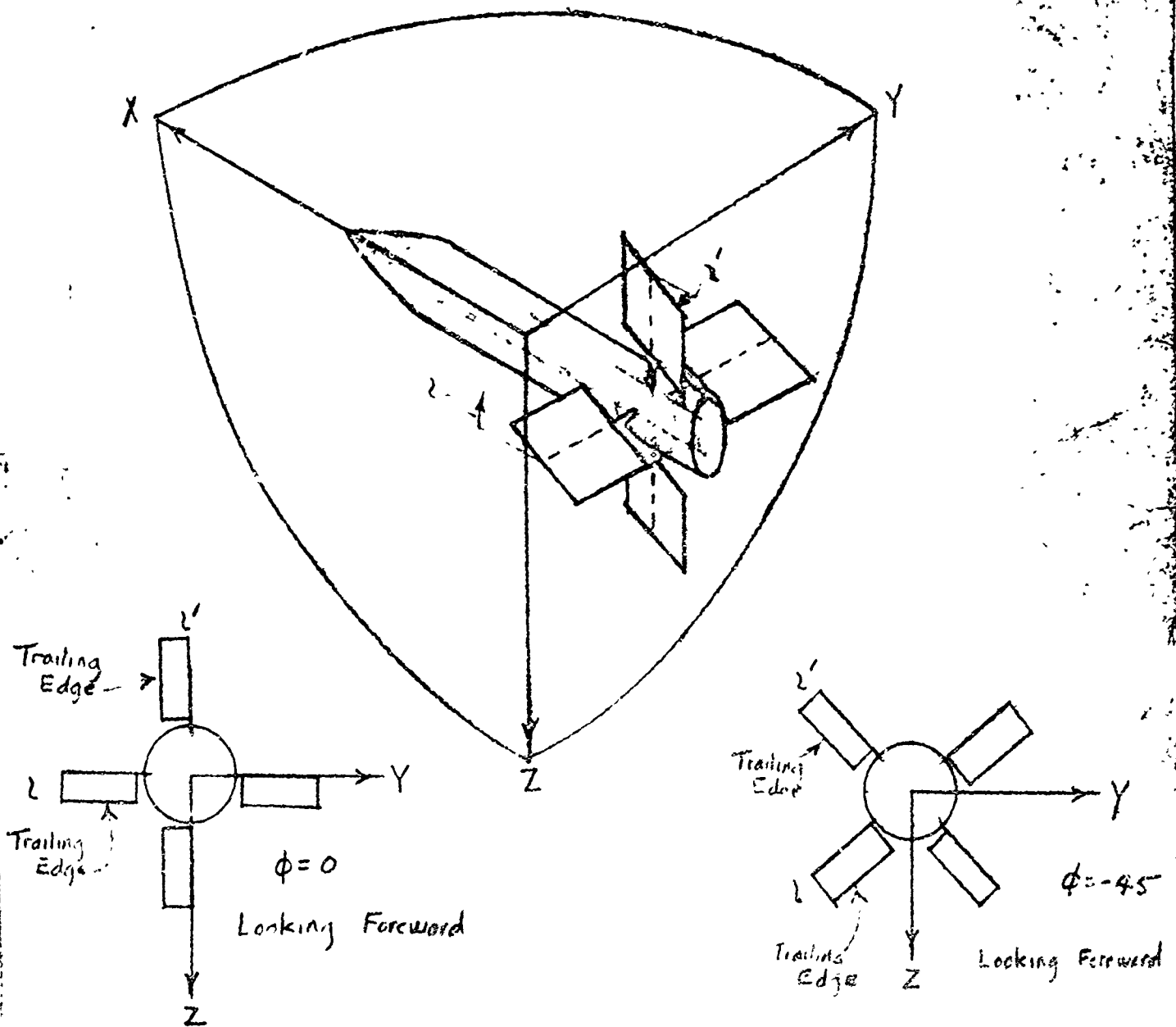
Correction Factor About Z Axis and the Locating Signal Area

Figure 3



Positive Rotation About Z Axis With Missile Initially at Angle of Attack α_0

Figure 4



Positive Reflections of Tail Surfaces

Figure 5

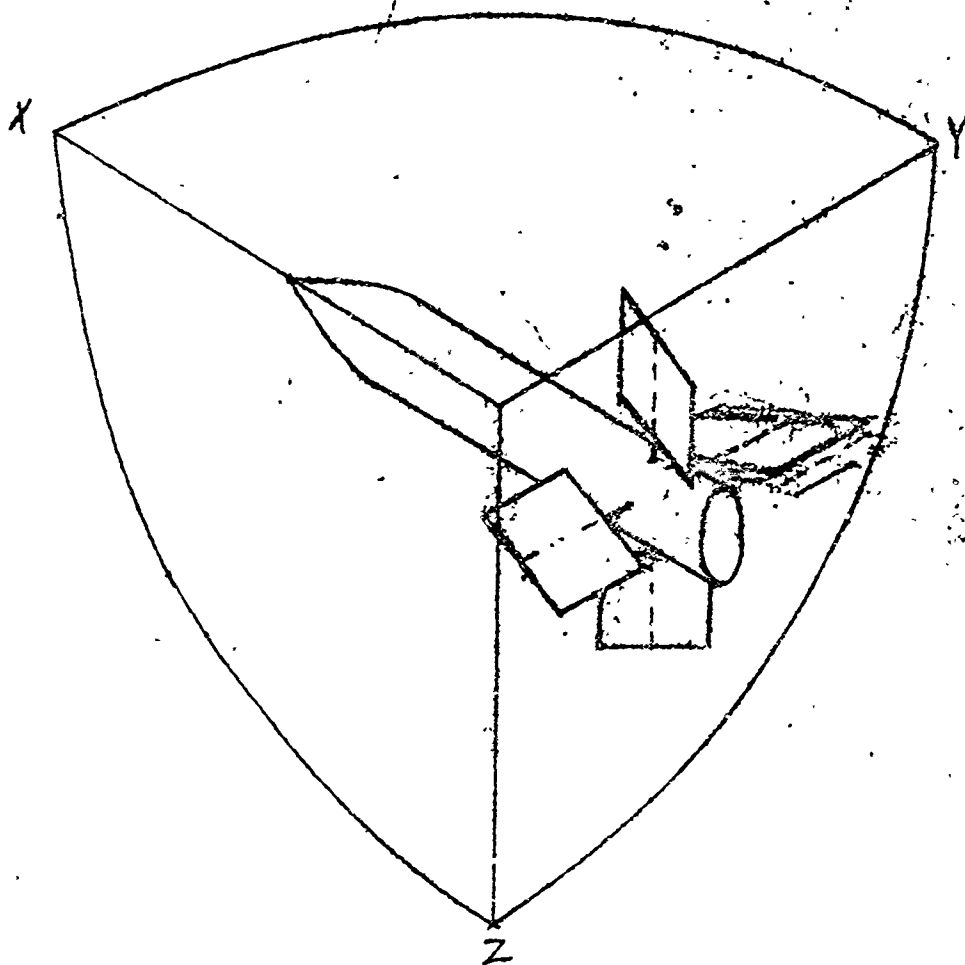


Diagram illustrating the geometry of the system

Figure 6